Economics Admissions Assessment 2016
Specimen Paper Section 1: explained answers

## PROBLEM SOLVING

1
The answer is option $\mathbf{C}$.
The maximum number of days in a calendar month is 31 .
Each day of the week occurs four times during the first 28 days of every month. Two of these days will be working Saturdays. If the 29th, 30th and 31st of a 31-day month are all weekdays, or if the 31st is a working Saturday, the number of working days during the month will be $(4 \times 5)+2+3=25$.

In a month when the neighbours work the maximum 25 days, one of them will drive 12 times and the other one will drive 13 times.

## 2

The answer is option $\mathbf{D}$.
The journey to work is 1800 m , which would take $1800 \div 5=360$ seconds $=6$ minutes if there was no delay at either of the sets of lights.

It takes $900 \div 5=180$ seconds $=3$ minutes to cycle from the first set of lights to the second. Because both sets change simultaneously, the maximum wait of 2 minutes at one of them will mean a wait of 1 minute at the other.

The longest journey time is therefore $6+2+1=9$ minutes.

## 3

The answer is option B.
There are eight possible side views of this paperweight: four with TLF uppermost (as shown in the question) and four with the reflection of TLF uppermost. You should visualise these views and eliminate the four options that are side views.

From the side, a vertical line will be seen at each boundary between a projection and a recess.
$\mathbf{A}$ and $\mathbf{E}$ are both views of the bases of the letters $T, L$ and $F$. $\mathbf{A}$ is the view when TLF is uppermost and $E$ is the view when the reflection of TLF is uppermost.
$\mathbf{C}$ is the view of the tops of the letters $\mathrm{T}, \mathrm{L}$ and F , with TLF uppermost.
D is the view of the edge of the letter F, with the reflection of TLF uppermost.
(Note that a view of the edge of the letter T would have a single vertical line one fifth of the way from one end.)

The answer is option $\mathbf{A}$.
The energy values in the table are for 100 g of each food. The energy provided by each of the first four ingredients is, therefore:

| mealworms | - | $1.5 \times 150=225$ calories |
| :--- | :--- | :--- |
| apples | - | $1.5 \times 350=525$ calories |
| raisins | - | $2.5 \times 300=750$ calories |
| suet | - | $1.25 \times 800=1000$ calories |

The total energy provided by these ingredients is $225+525+750+1000=2500$ calories.
This means that the sunflower seeds must provide 2500 calories in order to make up the required total of 5000 calories.

Sunflower seeds provide 500 calories per 100 g , so $(2500 \div 500) \times 100=500 \mathrm{~g}$ of sunflower seeds must be used.

## 5

The answer is option B.
The distance between Sue and Freya increases while Freya runs to collect the stick and decreases as she brings it back to Sue. Only options B and $\mathbf{D}$ show this situation.

If Sue were to stand still, it would take the same amount of time for Freya to run 20 m in both directions (assuming the same average speed both ways), but because Sue is walking towards her, the distance between them decreases from 20 m to 0 in a shorter time than it increases from 0 to 20 m .

## 6

The answer is option $\mathbf{E}$.
Because Alistair changes the code each time, you must not allow yourself to be distracted by the symbols making up his name. You simply need to identify which one of the options has repeated letters in the positions of the repeated symbols in the reply.

Only SOMETIME has the 7th letter the same as the 3rd letter and the 8th letter the same as the 4th letter.

The answer is option D.
The two journeys took a total of $24 / 32$ hours, which is 45 minutes. This means that on his second journey he travelled $24-15=9 \mathrm{~km}$ in $45-30=15$ minutes ( $1 / 4$ hour).

Speed $=$ distance $\div$ time, so his average speed on the second journey was $9 \mathrm{~km} \div 1 / 4$ hour $=36 \mathrm{~km} / \mathrm{h}$.

## 8

The answer is option $\mathbf{C}$.
This question requires you to extract and process the relevant data from both the table and the narrative.

You will need to compare the cost of silver membership for one year, which includes a free locker, with the cost of bronze membership for one year plus locker hire.

The cost of silver membership for one year is the renewal price of $£ 28$.
The cost of bronze membership for one year plus locker hire for 12 visits (one per month) is the renewal price of $£ 8+(12 \times £ 2)=£ 32$.

The cheaper option is silver membership for $£ 28$, which is $£ 12$ less than the $£ 40$ paid for gold membership 6 months ago as a new member.

## 9

The answer is option $\mathbf{A}$.

To answer this question you first need to establish that the number of 11-year olds who go swimming (104) is slightly greater than $2 / 3$ of the number of 16 -year olds who go swimming.

It is not necessary to carry out exact calculations for any of the sports; you only need to observe the following:

Football - $120 / 181$ is very close to $2 / 3$.
Cricket - 120/133 is considerably greater than $2 / 3$.
Hockey - 55/66 is considerably greater than $2 / 3$.
Tennis $\quad-\quad 123 / 149$ is considerably greater than $2 / 3$.
Squash $\quad-\quad 51 / 97$ is slightly greater than $1 / 2$.

## 10

The answer is option B.
The moving average of 13.82 beside 1913 means that $x+13.3+14.9+13.7+14.2=13.82 \times 5$, so $x+56.1=69.1$ and $x=13.0$.
$y$ is therefore $(13.4+14.4+13.4+13.7+13.0) \div 5=67.9 \div 5=13.58$.

## 11

The answer is option B.
A group of 10 students would only require 2 instructors, who would be paid $£ 12$ each for 2 hours, but the extra $£ 12$ for a third instructor is much less than the extra income from a group of 18 students.

A group of 18 students would pay a total of $18 \times(£ 7+£ 5)=£ 216$ for 2 hours, so the maximum profit the centre can make from a group after paying the instructors for a two-hour session is $£ 216-£ 36=$ £180.

## 12

The answer is option $\mathbf{E}$.
Grace has been late to lessons more than twice, so she can be eliminated.
Andrew and Edward have both failed to complete more than two pieces of homework by the deadline set, so they can also be eliminated.

Carole has 3 non-A-grade pieces of work, but lan has only 2 pieces of non-A-grade work, so lan will be awarded the prize.

## 13

The answer is option $\mathbf{B}$.
Because each group involves the same number of people, this question can be approached by assuming that 400 people were surveyed altogether (100 in each age group) and then analysing the five statements as follows:

In the age group $25-35$, 32 people preferred Drink A and 33 people preferred Drink B, so conclusion A cannot be drawn.

A total of 103 people preferred Drink B and 101 people had no preference, so conclusion B can be drawn.

A total of 48 people in the age group 5-15 expressed a preference. This is less than half of the people in the group, so conclusion $\mathbf{C}$ cannot be drawn.

A total of 225 people had a preference. This is more than half of the total number of people surveyed, so conclusion D cannot be drawn.

A total of 74 people didn't know which they preferred. $20 \%$ of $400=80$, so conclusion $\mathbf{E}$ cannot be drawn.

## 14

The answer is option $\mathbf{D}$.
The fact that the bills for April and November are different amounts for the same number of units used means that the original tariff applies to at least the first four months of the year.

From January to February (for instance) an increase of 25 units used increased the monthly charge by $£ 2.50$, so the original charge per unit was 10p per unit. From January to August the formula: $£ 30+$ (units used) $\times 10$ p consistently gives the monthly charge, but in September it doesn't.
(The tariff changed to $£ 20$ standing charge per month plus 20p per unit used in September, but it is not necessary to calculate this in order to answer the question.)

## 15

The answer is option $\mathbf{C}$.
The most efficient way of approaching this question is to solve the problem numerically. However, because it is the method of calculation that is important here and not the actual figures, time can be saved by using any number greater than 420 for the total number of desks available.

The total number of desks available in the 12 rooms (including the sports hall, though it doesn't make any difference to the method if you don't include it) is 456, so there are 36 spare desks (456-420). The exams officer wants to leave the same number of empty desks in each room, so he should divide 36 by 12 and subtract the answer (3) from the number of desks in each room. This is the method described in option C.

## 16

The answer is option $\mathbf{D}$.
The most efficient way of approaching this question is to work out your own answer from the graph and then select the option that is the closest.

A reasonable estimate for the cumulative rainfall at the beginning of June is 175 mm (three quarters of the way from 100 mm to 200 mm ) and it is clearly 400 mm at the end of September. This gives an average of $225 \div 4=56.25 \mathrm{~mm} /$ month for the 4 -month period. This is very close to $57 \mathrm{~mm} / \mathrm{month}$, which is option $\mathbf{D}$.

## 17

The answer is option $\mathbf{B}$.
The hot tap supplies 24 litres of water per minute ( 360 litres $\div 15$ minutes) and the cold tap supplies 36 litres of water per minute ( 360 litres $\div 10$ minutes). Together they supply a total of 90 litres in the $11 / 2$ minutes before the cold tap is turned off, which makes the bath $1 / 4$ full. The hot tap then supplies another 180 litres to make the bath $3 / 4$ full, taking a further $180 \div 24=7 \frac{1}{2}$ minutes.

In all five options the graph rises from $(0,0)$ to $(1,1)$ as the two taps together make the bath $1 / 4$ full in the first $1 \frac{1}{2}$ minutes. The correct graph then has to rise 2 units up the $y$-axis (from $1 / 4$ to $3 / 4$ ) as it advances 5 units along the $x$-axis ( $71 / 2$ minutes).

## 18

The answer is option $\mathbf{E}$.
The reflection of 22:05, and therefore the correct time when the watch was reset to 22:05, is 20:55. This means that the watch is 1 hour 10 minutes ahead of the correct time. As a result, when the watch displayed the time as 10:09 this morning, the view of the digital clock that could be seen was the reflection of 08:59.

## 19

The answer is option $\mathbf{A}$.
The bar chart reveals that the usage this year will be as follows (assuming it to be the same as last year, as instructed);

January - March: 6,000; April - June: 7,000; July - September: 5,000;
October - December: 6,000.
There were 2,000 containers left at the end of last year, so it would appear that only 4,000 need to be delivered in January. However, this would mean that there would be none left at the end of March, and, because the maximum number that can be delivered at one time is 6,000 , they would run out before the end of June.

Ordering 5,000 in January would raise the total in stock to 7,000 , with 1,000 left at the end of March. If 6,000 were then to be ordered in April there would be just enough to cover the usage of 7,000 from April to June. Following this, 5,000 could be ordered in July and 6,000 in October. This would result in there being none leftover at the end of the year.

## 20

The answer is option $\mathbf{E}$.
To answer this question you need to visualize the appearances of the trees from different parts of field Y . The most efficient approach is to imagine walking from one side of the field to the other.

Starting from the extreme top left (for example), the order of the trees seen changes as follows:
4, 3, 2, 1
4, 1 (with 3 behind), 2
4, 1, 3, 2
1 (with 4 behind), 3, 2
1, 4, 3, 2
1, 4, 2 (with 3 behind)
1, 4, 2, 3
1, 2 (with 4 behind), 3

## 1, 2, 4, 3

Photographs were only taken when all four trees were visible, so there can be five different orders in the photographs.

## MATHEMATICS

## 21

The answer is option B.
Square out the bracket and bring all terms to the numerator: $\left(9 x^{2}-12 x+4\right) x^{-\frac{3}{2}}$
Multiply out: $9 x^{\frac{1}{2}}-12 x^{-\frac{1}{2}}+4 x^{-\frac{3}{2}}$
Differentiate: $\frac{9}{2} x^{-\frac{1}{2}}+6 x^{-\frac{3}{2}}-6 x^{-\frac{5}{2}}$
Substitute $x=2: \frac{9}{2 \sqrt{2}}+\frac{6}{2 \sqrt{2}}-\frac{6}{4 \sqrt{2}}=\frac{12}{2 \sqrt{2}}=3 \sqrt{2}$

## 22

The answer is option E.
Because it's given that statement (*) is false, then it is not true that "every day next week, Fred will do at least one maths problem". Fred only needs to fail to do a maths problem on, say, Wednesday, for (*) to be false, even if he does maths problems every other day besides Wednesday.

## 23

The answer is option $\mathbf{F}$.
Take logs of each side and separate out the LHS:
$x \log _{10} a+2 x \log _{10} b+3 x \log _{10} c=\log _{10} 2$
$x\left(\log _{10} a+2 \log _{10} b+3 \log _{10} c\right)=\log _{10} 2$
$x \log _{10}\left(a b^{2} c^{3}\right)=\log _{10} 2$, so $x=\frac{\log _{10} 2}{\log _{10}\left(a b^{2} c^{3}\right)}$

## 24

The answer is option $\mathbf{A}$.
Roots are given by: $x=\frac{11 \pm \sqrt{121-8 c}}{4}$
The difference in roots is $\frac{1}{2} \sqrt{121-8 c}=2$
Then we deduce $\sqrt{121-8 c}=4$
Square and rearrange: $121-8 c=16$, giving $8 c=105$, so $c=\frac{105}{8}$

## 25

The answer is option $\mathbf{C}$.
Using just the runners' initials for simplicity, and writing ">" to mean "beat", we are told that:
$\mathrm{F}>\mathrm{G}, \mathrm{H}>\mathrm{L}, \mathrm{L}>\mathrm{G}, \mathrm{R}>\mathrm{G}$
It immediately follows that $\mathrm{H}>\mathrm{G}$, so G must have come last.
Among the other four, we only know that $\mathrm{H}>\mathrm{L}$.
There are several ways to work out the number of orders from here:

- There are $4!=24$ ways to order the four runners. In half of them $H>L$, in the other half, $L>H$. So there are 12 orders with $\mathrm{H}>\mathrm{L}$.
- We could choose two out of the four spots for $H$ and $L$; there are $\binom{4}{2}={ }^{4} C_{2}=6$ ways to do this. Then $H$ and $L$ are fixed, and there are two ways to put $F$ and $R$ in the remaining two spots, giving $6 \times 2=12$ orders.
- We could list all of the possibilities, always placing H before L :
o HLFR HLRF HFLR HFRL HRLF HRFL
o FHLR FHRL FRHL RHLF RHFL RFHL
One just has to take care to ensure that every possibility is listed.


## 26

The answer is option $\mathbf{E}$.
Let $y=2^{x}$, then $y^{2}-8 y+15=0$. Solve to give $y=3$ or 5
$2^{x}=3$ implies that $x=\frac{\log _{10} 3}{\log _{10} 2}$. Similarly, the other solution is $\frac{\log _{10} 5}{\log _{10} 2}$
Sum of roots $=\frac{\log _{10} 3+\log _{10} 5}{\log _{10} 2}=\frac{\log _{10} 15}{\log _{10} 2}$

## 27

The answer is option $\mathbf{B}$.
As the polynomial has degree 5 , its derivative will have degree 4 so may have up to four real roots, and so our graph may have up to four stationary points.
The graph of $\frac{d y}{d x}$ either tends to $+\infty$ as $x \rightarrow \pm \infty$ or it tends to $-\infty$ for both; either way, it must cross the $x$-axis an even number of times. (Touching the axis without crossing results in a point of inflection, which we are not considering in this question.)

So the possibilities for the graph of $\frac{d y}{d x}$ to cross the $x$-axis, along with a description of the corresponding turning points for $y$, are:
[diagram not to scale]

$\min -\max -\min -\max$

$\max -\min -\max -\min$

$\min -\max$
max - min
or the graph of $\frac{d y}{d x}$ does not cross the $x$-axis at all, in which case there are no stationary points.
So possibility $\mathbf{B}$, one local minimum and two local maxima, is impossible.

## 28

The answer is option $\mathbf{E}$.
Statement 1 subtracts $a+b$ from both sides.
Statement 2 can be written as $(a-b)^{2} \geq 0$. This is always true.
Statement 3 can be false if $c$ is negative, for example $a=2, b=1, c=-1$

## 29

The answer is option $\mathbf{C}$.
To determine how many roots the equation has, we can investigate how many times the graph of $y=x^{4}-4 x^{3}+4 x^{2}-10$ intersects the $x$-axis. To help us do this we can find where the graph has turning points:

Differentiating gives: $4 x^{3}-12 x^{2}+8 x=0$
Solving this: $4 x\left(x^{2}-3 x+2\right)=4 x(x-1)(x-2)=0$
Giving $x=0,1,2$
The coordinates of the turning points are: $(0,-10),(1,-9),(2,-10)$.
From this, and knowing the general shapes of quartics, we can deduce that the curve will intersect the $x$-axis at two distinct places and so the original quartic must have two distinct real roots.

## 30

The answer is option $\mathbf{D}$.
The answer is $y=a x^{b}$
We can see this by taking logs of the equation and comparing it with the standard equation of a line, $Y=m X+c$

Taking logs gives:
$\log y=\log a+b \log x$
Comparing with $Y=m X+c$, we can see that plotting $\log y$ on the $Y$-axis and $\log x$ on the $X$-axis would give us a straight line with a gradient of $b$ and a $Y$-intercept of $\log a$

The answer is option $\mathbf{A}$.
Multiply out the bracket and integrate term by term: $\left[\frac{x^{3}}{3}-a x^{2}+a^{2} x\right]_{0}^{1}$ or integrate directly to get $\left[\frac{(x-a)^{3}}{3}\right]_{0}^{1}$

Substitute limits 0 and 1: $a^{2}-a+\frac{1}{3}$
Complete the square: $\left(a-\frac{1}{2}\right)^{2}+\frac{1}{12}$
Minimum value as $a$ varies is $\frac{1}{12}$

## 32

The answer is option $\mathbf{E}$.
The largest median will be if the middle number is as large as possible.
If the largest of the five numbers is $u$, then we will make the third number also $u$. Since the range is 20 , the smallest number is $u-20$. To make the median $u$ as large as possible, we want the second smallest number to be $u-20$ too.

So the five numbers are
$u-20, u-20, u, u, u$
Their sum is $5 u-40=0$ as the mean is 0 , so $u=8$, which is the largest possible median.
Thus the answer is $\mathbf{E}$.
The first part of the argument can be justified algebraically as follows:
Let the five numbers, in order from smallest to largest, be
$u-20, u-a, u-b, u-c, u$
where $20 \geq a \geq b \geq c \geq 0$. We need the sum to be 0 , and wish to make $u-b$ as large as possible.
The sum is $5 u-(20+a+b+c)=0$, so rearranging, we obtain
$u=4+\frac{1}{5}(a+b+c)$
This gives our median as
$u-b=4+\frac{1}{5}(a-4 b+c)$
We know that $b \geq c$, so we can rewrite the right-hand side in terms of $b-c \geq 0$ to get
$u-b=4-\frac{4}{5}(b-c)+\frac{1}{5} a-\frac{3}{5} c$
This will be maximum when $b-c=0$ (as small as possible), $a=20$ (as large as possible) and $c=0$ (as small as possible).

This gives $a=20, b=c=0$, which gives $u=4+\frac{1}{5}(20+0+0)=8$, and so the five numbers are indeed $-12,-12,8,8,8$ as we justified non-algebraically at the start.

## 33

The answer is option $\mathbf{D}$.
For real distinct roots the discriminant condition gives:
$(a-2)^{2}>4 a(-2)$ or $(a-2)^{2}-4 a(-2)>0$
$a^{2}+4 a+4>0$
$(a+2)^{2}>0$
This is true for all values of $a$ except -2

## 34

The answer is option $\mathbf{C}$.
$P \propto \frac{1}{Q^{2}}$ so $P=\frac{k}{Q^{2}}$ for some $k$
We can use multiplying factors to work out how each symbol changes; for instance, as $Q$ increases by $40 \%$ its new value is $1.4 Q$

For this question, we have:
$Q_{\text {new }}=1.4 Q$
Giving:
$P_{\text {new }}=\frac{k}{(1.4 Q)^{2}}=\frac{k}{1.96 Q^{2}}=\frac{1}{1.96} P$
We then notice:
$\frac{1}{1.96} \approx \frac{1}{2}$ and $\frac{1}{1.96}>\frac{1}{2}$
so $P_{\text {new }}$ is a little over $50 \%$ of $P$ and so we can deduce that $P$ has decreased by a little under $50 \%$.

## 35

The answer is option $\mathbf{E}$.
The probability that the first ball is red is $\frac{x}{x+y+z}$
The probability that the second ball is blue is $\frac{y}{x+y+z}$
The probability that the first ball is red and the second is blue is given by:
$\frac{x}{x+y+z} \times \frac{y}{x+y+z}=\frac{x y}{(x+y+z)^{2}}$

## 36

The answer is option $\mathbf{D}$.
The first, second, and fourth terms of the GP are $4,4 r$ and $4 r^{3}$, respectively.
Since the terms are also in arithmetic series, the common difference is constant, so we can write:
$4 r-4=4 r^{3}-4 r$
Factorising this: $4(r-1)=4 r\left(r^{2}-1\right)$
Dividing by $4(r-1)$ since $r \neq 1$ :
$1=r(r+1)$ giving $r^{2}+r-1=0$
Solving: $r=\frac{1}{2}(\sqrt{5}-1)$
Sum to infinity $=\frac{a}{1-r}=\frac{8}{3-\sqrt{5}}$
Rationalising this gives: $2(3+\sqrt{5})$

