Engineering Admissions Assessment 2016

Specimen Paper Section 1: explained answers

MATHEMATICS AND PHYSICS

1

The answer is option **A**.

We can write:

area of shape $=$ area of square $-$ area of semi-circle.

If we let the length of one side of the square be $2x$, so that the radius of the semi-circle is x, then this becomes:

$$100 = 4x^2 - \frac{1}{2}\pi x^2 = x^2\left(4 - \frac{\pi}{2}\right)$$

Rearranging:

$$x = \sqrt{\frac{200}{8-\pi}} = 10\sqrt{\frac{2}{8-\pi}}$$

So the length of one side of the square is $2x = 20\sqrt{\frac{2}{8-\pi}}$

2

The answer is option **C**.

The forces on the parachutist are 600 N downwards and 900 N upwards.

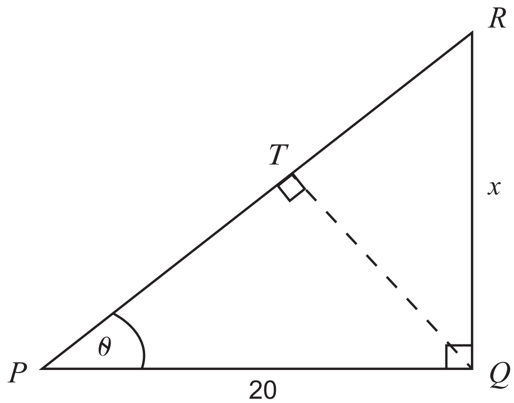
These cause a resultant force on the parachutist of 900 N $-$ 600 N $=$ 300 N upwards.

acceleration $=$ resultant force / mass $=$ 300 / 60 $=$ 5.0 m s^{-2} upwards.

3

The answer is option **E**.

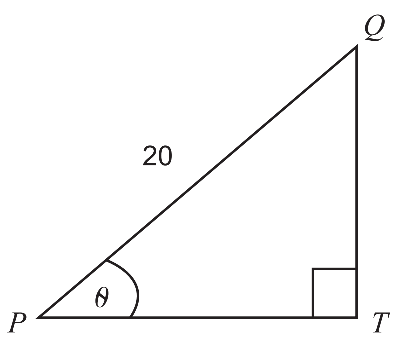
Let $RQ = x$

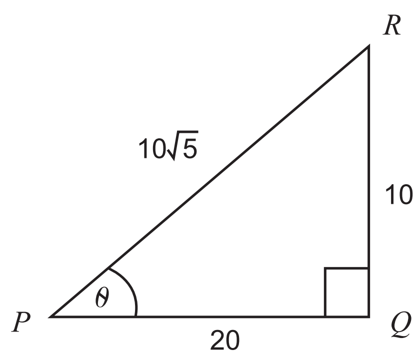


We know $RQ : PQ$ is $1:2$ so we can deduce $x = 10$

By Pythagoras' Theorem, we have $PR = 10\sqrt{5}$

We notice triangles PQT and PRQ are similar:





So we can write: $\dfrac{QT}{10} = \dfrac{20}{10\sqrt{5}}$

Giving: $QT = \dfrac{20}{\sqrt{5}} = 4\sqrt{5}$

4

The answer is option **C**.

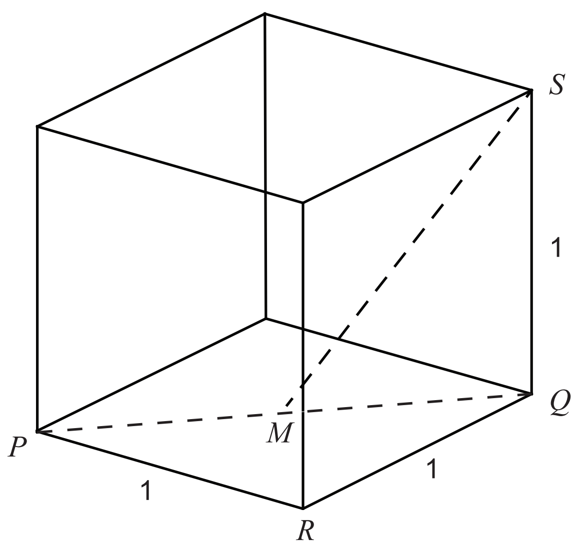
From the graph, period $T = 2.0\,\text{s}$

$f = \frac{1}{T} = \frac{1}{2.0} = 0.50\,\text{Hz}$

$v = f\lambda = 0.50 \times 1.5 = 0.75\,\text{cm}\,\text{s}^{-1}$

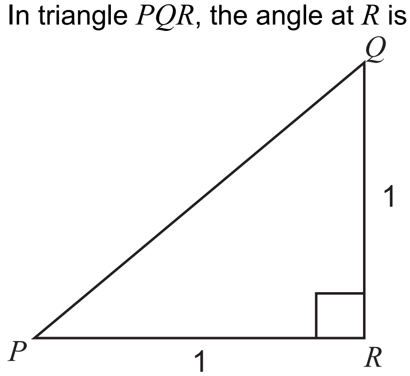
5

The answer is option **A**.



[diagram not to scale]

In triangle PQR, the angle at R is a right angle so we can use Pythagoras' Theorem to find PQ:

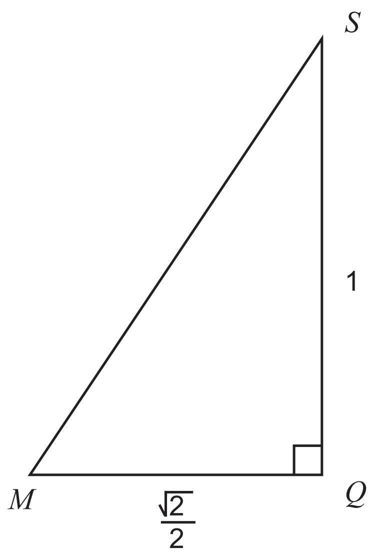


$PQ^2 = 1^2 + 1^2 = 2$

Giving $PQ = \sqrt{2}$

QM is half of PQ so $QM = \frac{\sqrt{2}}{2}$

Now looking at triangle MQS:



The angle at Q is a right angle so we can use Pythagoras' Theorem:

$$MS^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + 1^2 = \frac{2}{4} + 1 = \frac{3}{2}$$

Giving $MS = \sqrt{\frac{3}{2}}$

6

The answer is option **B**.

momentum $p = mv$ and kinetic energy $\mathrm{KE} = \frac{1}{2}mv^2$

rearranging:

$v = \frac{p}{m}$ and so $KE = \frac{1}{2}\frac{p^2}{m}$

therefore:

$m = \frac{p^2}{2KE} = \frac{900}{300} = 3.0\,\mathrm{kg}$

[Finally, it is good practice to verify this answer by substituting it back into the kinetic energy equation:

kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 10^2 = 150\,\mathrm{J}$, which is correct.]

7

The answer is option **D**.

When the minute hand is pointing to the 9 [indicating 45 minutes past the hour], the hour hand will have moved three-quarters of the angle between the 9 and the 10 on the clock dial. The angle between the 9 and the 10 is 30 degrees so the hour hand will have moved three-quarters of 30 degrees, or 22.5 degrees. The angle between the hands is, therefore, 22.5°.

8

The answer is option **E**.

Looking at each option in turn:

A $V = IR$ so volts are amp $\times$ ohm not amp per ohm.

B The volt is defined as a joule per coulomb.

Therefore (coulomb per joule) $=$ 1 / volt. This is not equal to the volt.

C The joule per second is the watt.

This is not equal to the volt.

D Since the volt is defined as a joule per coulomb it is worth expressing the newton as a joule per metre.

Therefore newton per coulomb $=$ (joule per metre) per coulomb $=$ (joule per coulomb) per metre $=$ volt per metre. This is not equal to the volt.

E $P = IV$, therefore $V = \frac{P}{I}$ so volts are watt per amp.

9

The answer is option **B**.

To answer this we shall use:
area of triangle $= \frac{1}{2} \times$ base $\times$ height
This gives:

$$\text{area} = \frac{1}{2}\left(4+\sqrt{2}\right)\left(2-\sqrt{2}\right) = \frac{1}{2}\left(8-4\sqrt{2}+2\sqrt{2}-\sqrt{2}\sqrt{2}\right) = \frac{1}{2}\left(6-2\sqrt{2}\right) = 3-\sqrt{2}$$

10

The answer is option **D**.

Consider each source in turn.

Source X: 24 hours is 5 half-lives (24 / 4.8 $=$ 5). So, in 24 hours the activity of source X will halve 5 times, from 320 to 160 to 80 to 40 to 20 to 10. After 24 hours the activity of X will have fallen to 10 Bq.

Source Y: 24 hours is 3 half-lives (24 / 8 $=$ 3). So, in 24 hours the activity of Y will halve 3 times, from 480 to 240 to 120 to 60. After 24 hours the activity of Y will have fallen to 60 Bq.

The combined count rate after 24 hours is therefore $10 + 60 = 70$ Bq.

11

The answer is option **D**.

As the cylinder has the same internal diameter as the sphere, it has the same radius, r. As the length of the cylinder is the same as the diameter of the sphere, it has length $2r$. So we can write:

Volume of the sphere $= \frac{4}{3}\pi r^3$

Volume of cylinder $=$ base area $\times$ height $= \pi r^2 \times 2r = 2\pi r^3$

This then allows us to calculate the fraction of the space in the cylinder taken up by the sphere:

$$\frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{2}{3}$$

12

The answer is option **E**.

As the cyclist loses 100 m of height, he loses gravitational potential energy

$= mgh$
$= 100 \times 10 \times 100$
$= 100\,000$ J

The cyclist is descending the slope at a constant speed, which means that all of this potential energy becomes heat as work is done on the cyclist by the resistive forces.

Therefore, work done by resistive forces $=$ 100 000 J

distance travelled along the slope whilst descending 100 m height $= 100 \times 10 = 1000$ m

Therefore, resistive force $=$ work done / distance $= 100\,000/1000 = 100$ N

Alternatively, since the cyclist is moving at a constant velocity he must be in equilibrium. Therefore the drag force must be equal and opposite to the component of weight acting down the slope ($mg\sin\theta$). In this case $\sin\theta = 10/100 = 0.1$ so the drag force is $0.1mg = 0.1 \times 100 \times 10 = 100$ N.

13

The answer is option **A**.

For $0 < x < 1$, we have:

$0 < x^2 < 1$

$0 < \sqrt{x} < 1$

and also:

$1 < \frac{1}{x}$

$\frac{1}{\sqrt{x}} < \frac{1}{x}$

$\frac{1}{1+x} < \frac{1}{x}$

so $\frac{1}{x}$ has the largest value in $0 < x < 1$

14

The answer is option **A**.

Let the mass of the ball in kg be *m*. (Its value is not needed to do the question.)

kinetic energy at the point of release $= \frac{1}{2}mv^2 = \frac{1}{2} \times m \times 12^2 = \frac{1}{2} \times 144 \times m = 72m$ joules

At the highest point, all of this kinetic energy has turned into gravitational potential energy

Therefore, at the highest point, gravitational potential energy $= 72m$ joules

This is also equal to *mgh*, where *h* is the height reached.

Therefore, $10mh = 72m$. Notice that *m* cancels out here (as expected), leaving $10h = 72$

Therefore, height reached $h = 72/10 = 7.2$ metres

Alternatively, since all of the initial KE $\left(\frac{1}{2}mv^2\right)$ is transferred to GPE (*mgh*) at the top of the motion:

$mgh = \frac{1}{2}mv^2$, so $h = v^2/2g = 12^2/20 = 7.2\,\text{m}$

15

The answer is option **C**.

Using the fact that triangle ABC is similar to triangle ADE [they have the same angles], we can write:

$$\frac{AB}{AD} = \frac{BC}{DE}$$

This gives:

$$\frac{4}{x} = \frac{x}{x+3}$$

From which we get:

$$4(x+3) = x^2$$

Rearranging:

$$x^2 - 4x - 12 = 0$$

Factorising:

$$(x-6)(x+2) = 0$$

Giving $x = 6$ [as we cannot have a negative value for x as it represents a length].

And so $DE = x + 3 = 6 + 3 = 9$

16

The answer is option **C**.

kinetic energy of lorry $= \frac{1}{2}mv^2$

As the lorry is moving along a horizontal road, the resistive forces need to absorb this energy and no more. Work done by resistive forces in stopping the lorry is therefore $\frac{1}{2}mv^2$

distance travelled by lorry while stopping $=$ work done / stopping force $= \left(\frac{1}{2}mv^2\right)/F = mv^2/2F$

17

The answer is option **C**.

$P \propto \frac{1}{Q^2}$ so $P = \frac{k}{Q^2}$ for some k

We can use multiplying factors to work out how each symbol changes; for instance, as Q increases by 40% its new value is $1.4Q$

For this question, we have:

$Q_{new} = 1.4Q$

Giving:

$$P_{new} = \frac{k}{(1.4Q)^2} = \frac{k}{1.96Q^2} = \frac{1}{1.96}P$$

We then notice:

$\frac{1}{1.96} \approx \frac{1}{2}$ and $\frac{1}{1.96} > \frac{1}{2}$

so P_{new} is a little over 50% of P and so we can deduce that P has decreased by a little under 50%.

18

The answer is option **B**.

An alpha particle consists of two protons and two neutrons so alpha emission results in the reduction of atomic number (bottom) by 2 and the reduction of nucleon number (top) by 4.

A beta-minus particle is an electron emitted when a neutron in the nucleus changes into a proton. This increases atomic number (bottom) by 1 but has no effect on nucleon number (since both protons and neutrons are nucleons).

The decay from X to Y involves the atomic number going down by 2 (from R to $R - 2$). This means that this stage must be alpha decay, and therefore that the mass number must drop by 4. The mass number of Y must therefore be $N - 4$, and this is the value of P.

The decay from Y to Z involves the mass number remaining the same (P in both cases). This means that this stage must be beta decay, and therefore that the atomic number of Z must be one higher than that of Y. As the atomic number of Y is $R - 2$, the atomic number of Z must be $(R - 2) + 1 = R - 1$, and this is the value of Q.

19

The answer is option **D**.

We are given: $x \propto z^2$ and $y \propto \frac{1}{z^3}$ which allows us to write: $x = kz^2$ and $y = \frac{c}{z^3}$

We want to eliminate z so we need both expressions to have the same power of z in them:

$x^3 = k^3 z^6$ and $y^2 = \frac{c^2}{z^6}$

Eliminating z and ignoring the constants gives:

$x^3 \propto \frac{1}{y^2}$

20

The answer is option **B**.

This question is simply a straightforward application of speed = distance / time, and the frequency of the pulse is irrelevant.

distance travelled by pulse from transmitter to foetus and back again to receiver $= 2 \times 10$
$= 20\,\text{cm}$
$= 0.20\,\text{m}$

Therefore, time taken = distance / speed = 0.20 / 500 = 0.0004 s = 0.40 ms

21

The answer is option **C**.

To make the question easier to answer, we can let $QX = 2$

We then find, using the given ratios, that $PX = 12$ and $XR = 3$ and so $PR = 15$

As M is the midpoint of PR then $PM = 7.5$ and so $MX = 4.5$

We can then calculate the ratio asked for:

$\frac{QX}{MX} = \frac{2}{4.5} = \frac{4}{9}$

22

The answer is option **B**.

Both graphs P and Q show an object accelerating with constant acceleration. The value of that acceleration is the gradient of the velocity-time graph.

Graph P: Gradient $=$ 10 / 24, which is not 2.4 m s^{-2}. (You don't need to evaluate 10 / 24 to see that.)

Graph Q: Gradient $=$ (58 $-$ 10) / 20 = 48 / 20 $=$ 2.4 m s^{-2}. So graph Q is correct.

Graphs R and S are both distance-time graphs which are straight lines, signifying constant speed. Constant speed means zero acceleration, and so neither of these graphs shows an acceleration of 2.4 m s^{-2}.

The correct answer is therefore that graph Q only shows an acceleration of 2.4 m s^{-2}.

23

The answer is option **D**.

$x^2 \geq 8 - 2x$

Rearranging:

$x^2 + 2x - 8 \geq 0$

Factorising:

$(x + 4)(x - 2) \geq 0$

Giving:

$x \leq -4$ or $x \geq 2$

24

The answer is option **D**.

Consider each statement in turn:

A Gamma emission is a form of naturally occurring radioactive decay. It is not the same process as nuclear fission. Statement not correct.

B The half-life of a substance is the time for half the amount of substance remaining at any moment to decay. A substance will never fully decay, and so ‘half the time taken to decay’ has no meaning. Statement not correct.

C The number of neutrons is the mass number minus the atomic number. Statement **C** has this the wrong way round. Statement not correct.

D When a nucleus emits a beta particle, a neutron turns into a proton (and an electron is emitted). The nucleus therefore contains the same number of particles as before, it’s just that one of them has changed into a different particle. Statement correct.

E A neutron becoming a proton and emitting an electron is beta decay, not alpha decay. Statement not correct.

25

The answer is option **A**.

We are told:

surface area of cylinder $=$ volume of cylinder

We can write this as follows:

$$2\pi r^2 + 2\pi rh = \pi r^2 h$$

Rearranging:

$$2r = rh - 2h$$

$$2r = h(r-2)$$

Giving:

$$\frac{2r}{r-2} = h$$

26

The answer is option **D**.

Total resistance of circuit = sum of separate resistances = $R_1 + R_2$

Current in circuit is therefore $I = \dfrac{V}{R_{\text{total}}}$

$$= \frac{V}{(R_1 + R_2)}$$

This current is the same in both resistors.

Voltage across R_1 is therefore $V = IR_1$

$$= \frac{V}{(R_1 + R_2)} \times R_1$$

$$= \frac{VR_1}{(R_1 + R_2)}$$

Therefore power dissipated in R_1 is $P = VI$

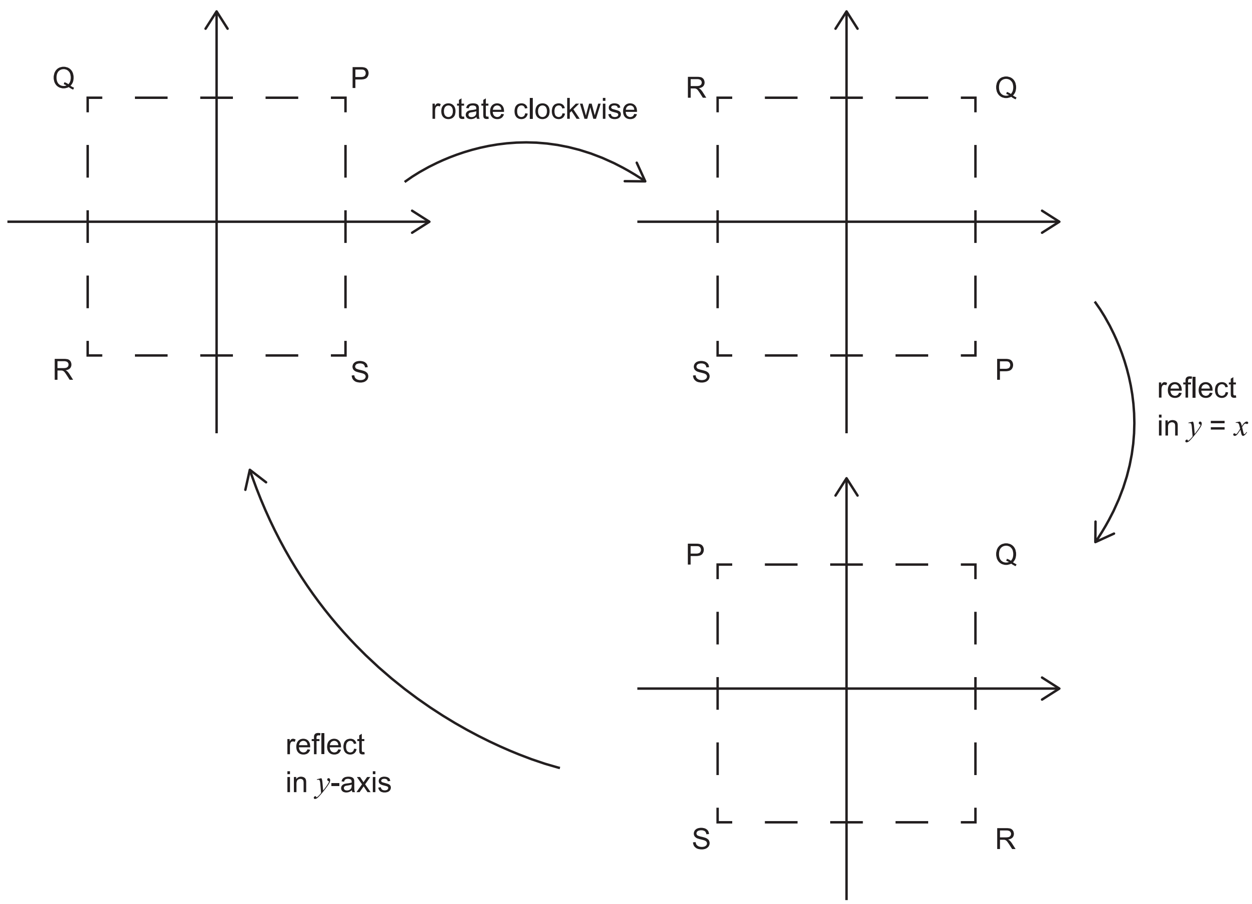
$$= \frac{VR_1}{(R_1 + R_2)} \times \frac{V}{(R_1 + R_2)}$$

$$= \frac{V^2 R_1}{(R_1 + R_2)^2}$$

Alternatively, a solution can be reached more quickly by considering dimensions: the power dissipated by a resistor is given by (voltage across resistor)^2 / (resistance). The only option with a voltage-squared divided by a resistance is **D**.

27

The answer is option **B**.



28

The answer is option **B**.

Consider each statement in turn:

P We have no information about the wavelength of the sound waves produced, and nor do we know how far the waves travel in a given time. It is therefore not possible to deduce the speed from the information given. Statement P cannot be deduced.

Q The maximum distance of oscillation from compression to rarefaction is 5.0 mm, but the amplitude of the wave is from the equilibrium position to the maxima. The amplitude is therefore half of this (2.5 mm) and statement Q is incorrect.

R We cannot deduce the wavelength from the information given, and hence cannot deduce statement R.

S The period of the pulses produced is 0.2 ms = 0.0002 s. This means the frequency of the wave is 1 / period = 1 / 0.0002 = 5000 Hz. Statement S therefore can be correctly deduced from the information given.

The correct answer is that statement S only can be correctly deduced.

ADVANCED MATHEMATICS AND ADVANCED PHYSICS

29

The answer is option **D**.

Factorise the numerator and denominator: $\dfrac{(x-2)(x+2)}{x(x-2)}$

Cancel the $(x-2)$ factor to leave $\dfrac{x+2}{x}$

30

The answer is option **A**.

$$\begin{aligned}\text{acceleration} &= \frac{\text{velocity change}}{\text{time taken}}\\ &= \frac{10}{5.0}\\ &= 2.0\,\text{m s}^{-2}\end{aligned}$$

$$\begin{aligned}\text{resultant force to produce this acceleration} &= \text{mass} \times \text{acceleration}\\ &= 1000 \times 2.0\\ &= 2.0\,\text{kN}\end{aligned}$$

$$\begin{aligned}\text{total of resistive forces} &= \text{force from engine} - \text{resultant force}\\ &= 3.0\,\text{kN} - 2.0\,\text{kN}\\ &= 1.0\,\text{kN}\end{aligned}$$

31

The answer is option **F**.

Take logs of each side and separate out the LHS:

$$x\log_{10} a + 2x\log_{10} b + 3x\log_{10} c = \log_{10} 2$$

$$x\left(\log_{10} a + 2\log_{10} b + 3\log_{10} c\right) = \log_{10} 2$$

$x\log_{10}\left(ab^2c^3\right) = \log_{10} 2$, so $x = \dfrac{\log_{10} 2}{\log_{10}\left(ab^2c^3\right)}$

32

The answer is option **C**.

Let the positive direction be the direction of initial motion of particle P.

momentum of particle P before the collision $= +(2 \times 3) = +6\,\text{kg m s}^{-1}$

momentum of particle Q before the collision $= -5 \times r$

Therefore, total momentum of the particles before the collision $= 6 - 5r$

momentum of particle P after the collision $= -(2 \times 1) = -2\,\text{kg m s}^{-1}$

momentum of particle Q after the collision $= +(\ \times 0.5r) = +2.5r$

Therefore, total momentum of the particles after the collision $= 2.5r - 2$

Therefore, because momentum is conserved in the collision:

$(6 - 5r) = (2.5r - 2)$

Rearranging:

$(6 + 2) = (2.5r + 5r)$

$8 = 7.5r$

$16 = 15r$

$r = \dfrac{16}{15}$

33

The answer is option **D**.

A: $\tan\left(\dfrac{3\pi}{4}\right) = -1$

B: $\log_{10} 100 = 2$

C: $\sin^{10} x \leq 1$ for any x; in this case, it equals 1

D: $\log_2 10 > \log_2 8 = 3$

E: $0 < \sqrt{2} - 1 < 1$ (as $1 < \sqrt{2} < 2$), so raising it to the power of 10 still gives a number less than 1

So the largest of the five numbers is **D**: $\log_2 10$

34

The answer is option **A**.

When travelling at terminal velocity, the force of air resistance (drag) on the parachutist will be equal and opposite to his weight. This is true when travelling at a high terminal velocity before opening the parachute and when travelling at a lower terminal velocity after opening the parachute. Terminal velocity means zero acceleration which means zero resultant force.

During the act of opening the parachute there is a rapid deceleration which means that momentarily there must be a resultant upwards force acting on the parachutist. During that instant there must therefore be an air resistance force which is much greater than the weight.

The correct graph is therefore one that shows the same air resistance force, in the same direction, throughout apart from during the moment of opening the parachute. And during that moment there must be a much larger air resistance force, again all in the same direction. The only graph that shows all this is graph **A**.

B and **D** are incorrect because they show the air resistance force changing direction, which it cannot do. It will always be upwards. **C** and **D** are incorrect because they show different air resistance forces at terminal velocity before and after opening the parachute.

35

The answer is option **E**.

Let $y = 2^x$, then $y^2 - 8y + 15 = 0$. Solve to give $y = 3$ or 5

$2^x = 3$ implies that $x = \dfrac{\log_{10} 3}{\log_{10} 2}$. Similarly, the other solution is $\dfrac{\log_{10} 5}{\log_{10} 2}$

Sum of roots $= \dfrac{\log_{10} 3 + \log_{10} 5}{\log_{10} 2} = \dfrac{\log_{10} 15}{\log_{10} 2}$

36

The answer is option **F**.

Graph X: This plots a quantity that starts off large and decreases to zero as terminal velocity is reached. This is true of both acceleration and resultant force, either of which could be graph X. Graph X cannot represent potential energy because on a horizontal road the potential energy will be constant.

Graph Y: This plots a quantity that starts off at zero and increases to a maximum value. This is true of the air resistance (drag force) acting on the car and is also true of the velocity of the car. Graph Y cannot represent the weight of the car, however, as this is constant.

Graph Z: This plots a quantity that remains constant throughout the acceleration to terminal speed. The weight of the car does remain constant but its kinetic energy increases from zero to a maximum value. Graph Z must therefore represent the weight of the car.

	X	Y	Z
F	resultant force	velocity	weight of car

37

The answer is option **E**.

Statement 1 subtracts $a+b$ from both sides.

Statement 2 can be written as $(a-b)^2 \geq 0$ .This is always true.

Statement 3 can be false if c is negative, for example $a = 2$, $b = 1$, $c = -1$

38

The answer is option **C**.

The friction acting on the block cannot exceed the applied force. At first the friction cancels the applied force and the block is stationary. When the friction reaches its maximum magnitude, the net force is zero but as the applied force continues to increase, the block moves. The net force is increasing as the applied force increases and so the block accelerates with an increasing acceleration.

Only the options where the block remains stationary at first can be possible. In two of these cases, once the block moves, it moves with either constant velocity (zero acceleration) or with constant acceleration.

The correct statement is: It remains stationary at first and then accelerates forwards with an increasing acceleration.

39

The answer is option **A**.

The sequence begins $a_1 = 2$, $a_2 = 2 + (-1) = 1$, $a_3 = 2 = 1 + (-1)^2 = 2$, $a_4 = 1$, etc.

So the sum of the first 100 terms is $2 + 1 + 2 + 1 + \ldots = 50 \times 2 + 50 \times 1 = 150$

40

The answer is option **C**.

Use the conservation of momentum to calculate the speed of the second ball.

momentum before $= 0.2 \times 3.0 = 0.6$ kg m s^{-1}

Let the speed of the second ball after the collision be v ms^{-1}

total momentum after collision $= (0.2 \times 1.0) + (0.2v)$ kg m s^{-1}

total momentum after collision $=$ total momentum before collision

$(0.2 \times 1.0) + (0.2v) = 0.6$ kg m s^{-1}

$0.2v = 0.6 - 0.2 = 0.4$

$v = \dfrac{0.4}{0.2} = 2.0$ m s^{-1}

41

The answer is option **C**.

To determine how many roots the equation has, we can investigate how many times the graph of $y = x^4 - 4x^3 + 4x^2 - 10$ intersects the x-axis. To help us do this we can find where the graph has turning points:

Differentiating gives: $4x^3 - 12x^2 + 8x = 0$

Solving this: $4x(x^2 - 3x + 2) = 4x(x-1)(x-2) = 0$

Giving $x = 0$, 1, 2

The coordinates of the turning points are: (0, – 10), (1, – 9), (2, – 10).

From this, and knowing the general shapes of quartics, we can deduce that the curve will intersect the x-axis at two distinct places and so the original quartic must have two distinct real roots.

42

The answer is option **B**.

initial kinetic energy $= \frac{1}{2}mv^2$

$= mg_ph$ at maximum height

mass *m* can be cancelled

so $\frac{1}{2}v^2 = g_ph$

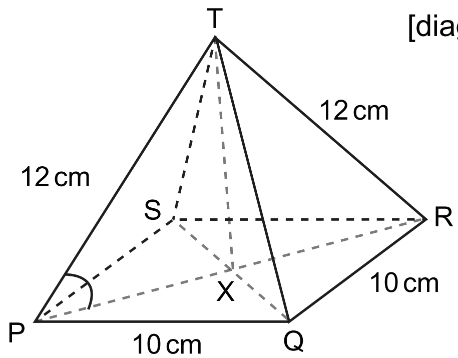
since $v = 20$, $200 = g_ph$

only one option has 200 as the product of g_p and h, $g_p = 5.0$; $h = 40$

43

The answer is option **F**.

1.

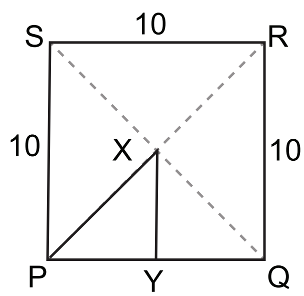


[diagram not to scale]

We need to find angle TPX.

Draw a perpendicular from T to X (midpoint of square PQRS), then find PX.

2.



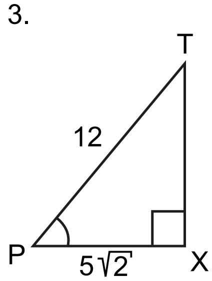
[diagram not to scale]

Using Pythagoras' Theorem

$PX^2 = 5^2 + 5^2 = 50$

so $PX = \sqrt{50} = 5\sqrt{2}$

3.



[diagram not to scale]

Then $\cos TPX = \frac{5\sqrt{2}}{12}$, so $TPX = \cos^{-1}\frac{5\sqrt{2}}{12}$

44

The answer is option **A**.

Initial potential energy of the ball $= mgh = 16mg$

Kinetic energy just before first bounce is therefore $16mg$

Kinetic energy just after first bounce $= \frac{1}{2} \times 16mg$

Therefore height of rebound after first bounce $= \frac{1}{2} \times 16 = 8\,\text{m}$

The height of rebound halves each time.

So height of rebound after second bounce $= 4\,\text{m}$

So height of rebound after third bounce $= 2\,\text{m}$

So height of rebound after fourth bounce $= 1\,\text{m} = 100\,\text{cm}$

It is after the fourth bounce, therefore, that the height of rebound first falls below 160 cm, so the 4$^{\text{th}}$ bounce is the correct answer.

45

The answer is option **D**.

The answer is $y = ax^b$

We can see this by taking logs of the equation and comparing it with the standard equation of a line, $Y = mX + c$

Taking logs gives:

$\log y = \log a + b \log x$

Comparing with $Y = mX + c$, we can see that plotting $\log y$ on the Y-axis and $\log x$ on the X-axis would give us a straight line with a gradient of b and a Y-intercept of $\log a$

46

The answer is option **E**.

The fact that the reading on the scales has decreased means that the normal contact force acting on the man has decreased. As his weight is now greater than the normal contact force there is a resultant force acting on the man which is downwards. This in turn means that the man has a downwards acceleration. This could mean either that he is moving upwards but slowing down or moving downwards and speeding up. The first of these is the one to appear in the options, and the correct answer is that the elevator is moving upwards with decreasing speed.

47

The answer is option **D**.

For real distinct roots the discriminant condition gives:

$(a-2)^2 > 4a(-2)$ or $(a-2)^2 - 4a(-2) > 0$

$a^2 + 4a + 4 > 0$

$(a+2)^2 > 0$

This is true for all values of a except -2

48

The answer is option **B**.

The deceleration to rest is constant and so the average velocity is $\left[\dfrac{(12.0+0)}{2}\right]$ m s^{-1} $= 6.0$ m s^{-1}.

Secondly, acceleration $= \dfrac{\text{velocity change}}{\text{time taken}}$, so:

time taken $= \dfrac{\text{velocity change}}{\text{acceleration}} = \dfrac{12.0}{1.5} = 8.0$ s to come to rest.

In 8.0 s at an average velocity of 6.0 m s^{-1} along a straight, horizontal track, the tram travels a distance of (6.0×8.0) m. This gives an answer of 48.0 m.

Alternatively, using $v^2 = u^2 + 2as$

Re-arranging:

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 12^2}{2 \times (-1.5)} = \frac{-144}{-3} = 48.0\text{ m}$$

49

The answer is option **C**.

If the three (integer) sides are equal there is the 4-4-4 (equilateral) triangle.

If the largest side is 5 cm, then we have two essentially different (non-congruent) triangles possible:

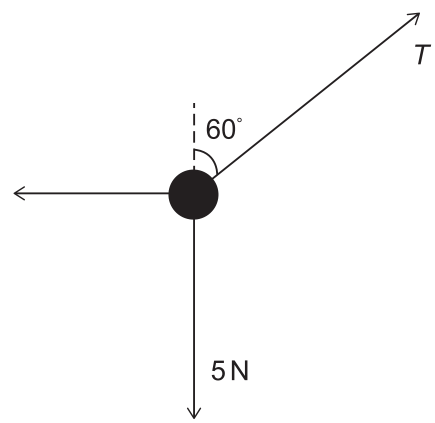
a (right-angled) 3-4-5 triangle (noting that the order of sides is unimportant); and a 2-5-5 (isosceles) triangle.

However, in every other combination of numbers the longest side is exceeded in length by the sum of the two shorter sides. If you imagine constructing the 'triangle' by drawing the long side first and then the shorter sides, one at each end of the longest one, then the shorter sides would not meet. So there cannot be a triangle with a side greater than or equal to 6.

50

The answer is option **C**.

You will find it helpful to draw a force diagram to help you to visualise the information given in this question:



T is the tension in the diagonal rope.

By resolving forces in the vertical direction:

$T\cos 60° = 5$

and resolving forces in the horizontal direction:

tension in horizontal rope $= T\sin 60°$

Since $\cos 60°$ is 0.5, we can see that $0.5T = 5$, and so $T = 10$

This means the tension in the horizontal rope $= 10\sin 60° = 10 \times \left(\frac{\sqrt{3}}{2}\right) = 5\sqrt{3}$

51

The answer is option **F**.

Since lengths are given to the nearest cm, the minimum possible length of PQ and the maximum possible length of PR are 3.5 and 2.5, respectively.

Use Pythagoras' Theorem: $\mathrm{QR}^2 = 3.5^2 - 2.5^2 = 12.25 - 6.25 = 6$

Take square root: $\mathrm{QR} = \sqrt{6}$

52

The answer is option **D**.

At 10 m, the velocity of the mass is constant.

Power is the rate at which work is done. In this case the work done (W) by the crane is the weight lifted (mg), multiplied by the height the weight was raised (h), so the work done in lifting the load to height h is $W = mgh$.

Differentiate this to find the rate of doing work:

$$P = \frac{dW}{dt} = mg\left(\frac{dh}{dt}\right)$$

$\frac{dh}{dt}$ is the gradient of the graph so the power output is equal to (mg) $\times$ gradient

When the height is 10 m the gradient is: $(17.5 - 2.5)\,\text{m}/(40 - 10)\,\text{s} = 15\,\text{m}/30\,\text{s} = 0.50\,\text{m}\,\text{s}^{-1}$

The power output is therefore $20 \times 10 \times 0.50 = 100\,\text{W}$

53

The answer is option **B**.

$-1 \le \tan x \le 1$ is satisfied when $0 \le x \le \frac{\pi}{4}$ and $\frac{3\pi}{4} \le x \le \pi$

$\sin 2x \ge 0.5$ is satisfied when $\frac{\pi}{6} \le 2x \le \frac{5\pi}{6}$ so $\frac{\pi}{12} \le x \le \frac{5\pi}{12}$

So both are satisfied together when $\frac{\pi}{12} \le x \le \frac{\pi}{4}$

The length of the interval $= \frac{\pi}{6}$

54

The answer is option **A**.

First, consider the whole train:

mass of train $= 20\,000 + 5000 + 5000 = 30\,000\,\text{kg}$

The only horizontal force acting on the train is the 15 000 N thrust force from the engine, so this is also the resultant force acting on the train.

Acceleration of train is therefore:

resultant force / mass $= \dfrac{15\,000}{30\,000} = 0.50\,\text{m}\,\text{s}^{-2}$

Now consider carriage 2. Carriage 2 is, in common with the rest of the train, accelerating at $0.50\,\text{m}\,\text{s}^{-2}$

The only horizontal force acting on carriage 2 is the tension T in its coupling with carriage 1.

Therefore, T = mass $\times$ acceleration $= 5000 \times 0.50 = 2500\,\text{N}$