UNIVERSITY OF CAMBRIDGE

Engineering Admissions Assessment

Section 2

Specimen: Explained Answers

1

The answer is option **C**.

This problem can be solved using the period and speed of the wave.

Re-arrange the equation *wave speed* (v) = *frequency* (f) × *wavelength* (λ) to give

$$\lambda = \frac{v}{f}$$

The period of a wave is the time taken for a particle of the medium through which the wave is travelling to perform one complete oscillation.

$$f = \frac{1}{\text{period}}$$

$$\lambda = v \times \text{period}$$

The time taken to move from equilibrium position to maximum displacement is 0.2 s, and this is one quarter of the period. The wave speed is 60 cm s^{-1}.

$$\lambda = 60\,\text{cm s}^{-1} \times 4 \times 0.2\text{ s} = 48\,\text{cm}$$

2

The answer is option **D**.

This problem can be solved by considering the change in height and the motion along the slope.

Both blocks fall through the same vertical height h so they transfer the same amount of gravitational potential energy to kinetic energy:

change in gravitational potential energy = change in kinetic energy

$$mgh = \frac{1}{2}mv^2$$

where m is the mass, g is the gravitational field strength and v is the final velocity, since both blocks start from rest.

Both blocks therefore reach the same final velocity:

$$v_X = v_Y$$

Both blocks accelerate down the slopes at a constant rate so their average velocities are also the same:

$$\frac{v_X}{2} = \frac{v_Y}{2}$$

However, X travels a longer distance along the slope than Y so takes a longer time to travel through the same vertical height:

$$t_X > t_Y$$

Therefore $t_X > t_Y$ and $v_X = v_Y$

3

The answer is option **G**.

This problem can be solved by considering the conservation of momentum and the conservation of energy.

The probe is travelling through the vacuum of deep space, its engine is switched off and there are no gravitational forces acting on it. Therefore it is an isolated system and the total momentum before the collision will equal the total momentum afterwards.

The fuel explodes and so its chemical energy decreases but since energy must be conserved and the system is isolated, some or all of the energy is transferred to kinetic energy and so increases the total kinetic energy.

Therefore the correct answer is the total momentum after the explosion is the same as before but the total kinetic energy has increased.

4

The answer is option **A**.

This problem can be solved by using conservation of energy and work done.

The kinetic energy (KE) lost by the object is equal to the gain in gravitational potential energy (GPE) plus the work done against the frictional force (W).

$$\text{gain in GPE} = mgh = 5 \times g \times 3 = 150\,\text{J}$$

$$W = Fd$$

where F is the frictional force and d is the distance travelled by the object

$$\text{length PQ} = d$$

From the diagram, $\sin\alpha = \frac{3}{d}$ so PQ = $d = \frac{3}{\sin\alpha}$

Since $\tan\alpha = \frac{3}{4}$, this is a 3,4,5 triangle and so:

$$\sin\alpha = \frac{3}{5} = 0.6$$

$$d = \frac{3}{0.6} = 5\,\text{m}$$

$$W = 5F$$

$$\text{change in KE} = \text{GPE gained} + W$$

$$210 = 150 + 5F$$

$$F = \frac{210 - 150}{5}$$

$$F = 12\,\text{N}$$

5

The answer is option **C**.

This problem can be solved by considering the vertical and horizontal motion separately and using the equations of motion, since the acceleration is constant.

Vertically:

$$s = ut + \tfrac{1}{2}at^2$$

$$4 = \tfrac{1}{2}gt^2$$

$$t^2 = \frac{4}{5}$$

$$t = \frac{2}{\sqrt{5}}\,\text{s}$$

Horizontally:

$$\text{distance} = \text{speed} \times \text{time}$$

$$\frac{6\sqrt{5}}{5} = \frac{2\sqrt{5}}{5}v$$

$$v = \frac{5 \times 6\sqrt{5}}{5 \times 2\sqrt{5}}$$

$$v = 3\,\text{m}\,\text{s}^{-1}$$

6

The answer is option **A**.

This problem can be solved by using $V = IR$ and resistivity = resistance $\times \dfrac{\text{cross-sectional area}}{\text{length}}$, $\rho = \dfrac{RA}{l}$

The current in the resistor is given by:

$$I = \frac{V}{R} = \frac{\text{pd across resistor}}{\text{resistance of resistor}} = \frac{1.0}{1.0 \times 10^{3}} = 1.0 \times 10^{-3}\,\text{A}$$

The wire and the resistor are connected in series so the current is the same in each of them.

The wire and the resistor act as a potential divider. The 1.2 V across the arrangement is shared between the wire and the resistor.

There is 1.0 V across the resistor so the pd across the wire is:

$$(1.2 - 1.0) = 0.20\,\text{V}$$

Therefore the resistance of the wire is given by:

$$R = \frac{V}{I} = \frac{\text{pd across wire}}{\text{current in wire}} = \frac{0.2}{1.0 \times 10^{-3}} = 2.0 \times 10^{2}\,\Omega$$

The resistivity is then:

$$\rho = \frac{RA}{l} = \frac{(2.0 \times 10^{2}) \times (0.020 \times 10^{-6})}{4.0} = 1.0 \times 10^{-6}\,\Omega\,\text{m}$$

7

The answer is option **B**

This problem can be solved by applying Newton's second law to each block.

The forces acting on the 6 kg block are its weight 60 N vertically downwards and the tension T N in the string, vertically upwards.

The forces acting on the 4 kg block are the tension in the string, upwards along the plane, a component of the weight, $40\sin 30°$ downwards along the plane and the frictional force F. Given the masses of the blocks, the acceleration of the 4 kg block must be upwards along the plane, and F must act downwards along the plane.

The system moves with constant acceleration, a, upwards along the plane.

The acceleration of the 4 kg block is the same as the acceleration of the 6 kg block.

Applying Newton's second law to each block:

$$60 - T = 6a \quad \text{and} \quad T - 15 - 40\sin 30° = 4a$$

This gives two simultaneous equations in T and a:

$$60 - T = 6a$$
$$T - 35 = 4a$$

Adding these gives:

$$10a = 25$$
$$a = 2.5$$

The acceleration of each of the blocks is $2.5\,\text{m s}^{-2}$.

8

The answer is option **A**.

This problem can be solved by considering each set of resistors and the internal resistance separately.

The combined resistance of the two $10\,\Omega$ resistors in parallel is given by:

$$\frac{1}{R_{\text{total}}} = \frac{1}{10\,\Omega} + \frac{1}{10\,\Omega} = \frac{1}{5\,\Omega}$$

$$R_{\text{total}} = \frac{10\,\Omega \times 10\,\Omega}{10\,\Omega + 10\,\Omega}$$

$$R_{\text{total}} = 5.0\,\Omega$$

The total resistance in the circuit, not including the internal resistance of the battery, is:

$$5.0\Omega + 3.0\Omega = 8.0\Omega$$

The total resistance in the circuit including the internal resistance of the battery is:

$$\frac{\text{emf}}{I} = \frac{20\,\text{V}}{2.0\,\text{A}} = 10\,\Omega$$

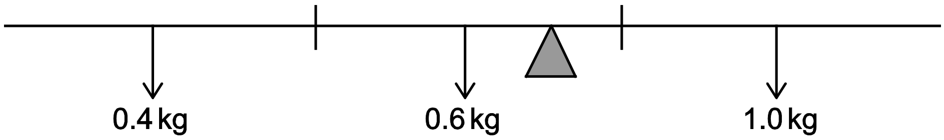
The internal resistance of the battery is $10\Omega - 8.0\Omega = 2.0\Omega$

9

The answer is option **C**

This problem can be solved by taking moments about the support.

The forces on the telescope are the weights of each of the three tubes, each acting at the midpoint of the tubes, because the tubes are uniform. Each tube has a length of 20 cm.



There is a normal contact force at the support, vertically upwards, but this calculation is not required and the problem can be solved by taking moments about the support. For the system to be in equilibrium, the support must be closer to the centre of the 1.0 kg tube than the centre of the 0.4 kg tube, as shown in the diagram.

The support is at an unknown distance x cm from the centre of the 0.6 kg tube, weight = 6 N

The moment of the 4 N weight is $4 \times (20 + x)$ anticlockwise

The moment of the 6 N weight is $6x$ anticlockwise

The moment of the 10 N weight is $10 \times (20 - x)$ clockwise.

For the telescope to remain horizontal, anticlockwise and clockwise moments must balance:

$$4(20 + x) + 6x = 10(20 - x)$$

$$80 + 10x = 200 - 10x$$

$$20x = 120$$

$$x = 6\text{ cm}$$

The support is 20 cm + 10 cm + 6 cm = 36 cm from the lighter end.

10

The answer is option **C**.

This problem can be solved by first considering one cable individually.

Calculate the force applied by one of the cables by combining the equation for Young modulus:

$$E = \frac{\sigma}{\varepsilon}$$

and the equation defining stress:

$$\sigma = \frac{F}{A}$$

to give:

$$\sigma = \varepsilon E = \frac{F}{A}$$

$$F = \varepsilon E A$$

$$F = 0.0025 \times 2.0 \times 10^{11} \times 2.0 \times 10^{-4}$$

$$F = 1.0 \times 10^{5}\,\mathrm{N}$$

The four cables are connected in parallel so the total force on the boulder is $4.0 \times 10^{5}\,\mathrm{N} = 400\,\mathrm{kN}$.

The boulder is moving at constant velocity so the resultant force is zero and the frictional force is equal to the total pulling force. Therefore the magnitude of the frictional force is also 400 kN.

Power transfer is given by $P = Fv = 400\,000\,\mathrm{N} \times 0.20\,\mathrm{m\,s^{-1}} = 80\,000\,\mathrm{W} = 80\,\mathrm{kW}$.

11

The answer is option **E**.

This problem can be solved by considering the phase difference as a fraction of a full cycle of vibration.

One cycle of vibration causes a phase change of 2π.

The phase difference between the vibrations at the two buildings is $\pi/3$, which is equivalent to $\frac{1}{6}$ of a cycle.

Therefore the 1.0 km distance between the buildings must be $\frac{1}{6}$ of a wavelength and so the wavelength of the seismic wave is 6.0 km.

The frequency of the seismic wave $f = \frac{1}{\text{period}} = \frac{1}{2.0}$

The speed of the seismic wave is given by:

$$v = f\lambda = \frac{6.0}{2.0} = 3.0\,\text{km}\,\text{s}^{-1}$$

12

The answer is option **B**.

This problem can be solved by considering the situation when the object is falling at an acceleration of $0.5g$ with velocity v_0.

Weight of object $= mg$

Drag force acting on object $F = kv_0^n$

Resultant force on object downwards $= mg - kv_0^n = ma = 0.5mg$

Rearranging:

$$\frac{1}{2}mg = kv_0^n$$

$$k = \frac{mg}{2v_0^n}$$

Now consider the object falling at terminal speed v_T:

Drag force = weight, so:

$$kv_T^n = mg$$

$$\frac{mg}{2v_0^n} v_T^n = mg$$

$$2v_0^n = v_T^n$$

Raising both sides to the power $(1/n)$:

$$2^{(1/n)} v_0 = v_T$$

13

The answer is option **E**.

This problem can be solved by considering the ratio of the forces on and extensions of the springs.

The weight of each mass is $0.10\,\text{kg} \times 10\,\text{N}\,\text{kg}^{-1} = 1.0\,\text{N}$

The weight of the springs can be ignored because their masses are negligible.

The force on the upper spring is 2.0 N, and the force on the lower spring is 1.0 N.

As they are subjected to different forces, the two springs will have different extensions.

Force, F, and extension, x, are related by $F = kx$ and the spring constant, k, is the same for both springs.

Therefore, since the ratio of the forces on the upper and lower springs is 2 : 1, their extensions must also be in the ratio 2 : 1

The sum of the extensions of the two springs is $30.0\,\text{cm} - (2 \times 12.0\,\text{cm}) = 6.0\,\text{cm}$.

Splitting this in the ratio 2 : 1 gives 4.0 cm : 2.0 cm for the extensions of the upper and lower springs respectively.

The spring constant k can be calculated by considering either of the springs:

Upper spring:

$$k = \frac{F}{x} = \frac{2.0\,\text{N}}{4.0\,\text{cm}} = 0.50\,\text{N}\,\text{cm}^{-1}$$

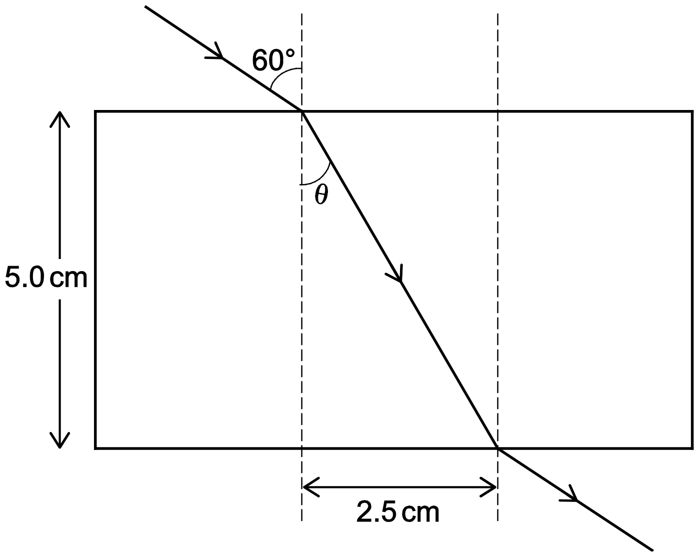
Lower spring:

$$k = \frac{F}{x} = \frac{1.0\,\text{N}}{2.0\,\text{cm}} = 0.50\,\text{N}\,\text{cm}^{-1}$$

14

The answer is option **H**.

This problem can be solved by using trigonometry to find the angle θ (shown in the diagram) and then using the law of refraction to find the refractive index n of the block.



Using Pythagoras, $\sin\theta = \dfrac{2.5}{\sqrt{2.5^2 + 5.0^2}} = \dfrac{2.5}{\sqrt{\frac{25}{4} + 25}} = \dfrac{2.5}{\sqrt{\frac{125}{4}}} = \dfrac{2.5}{\sqrt{\frac{5 \times 5 \times 5}{4}}} = \dfrac{2.5}{2.5\sqrt{5}} = \dfrac{1}{\sqrt{5}}$

Since $\sin 60° = \frac{\sqrt{3}}{2}$

$$n = \frac{\sin 60°}{\sin\theta} = \frac{\sqrt{3}/2}{1/\sqrt{5}} = \frac{\sqrt{3}\sqrt{5}}{2} = \frac{\sqrt{15}}{2}$$

15

The answer is option **B**

This problem can be solved by considering the change in kinetic energy and the conservation of momentum.

Let the speeds (in m s^{-1}) of the spheres after the collision be u and v respectively, both in the direction of motion of the 3 kg sphere before the collision.

Using the kinetic energy KE $= \frac{1}{2}mv^2$, the total kinetic energy of the spheres are:

after the collision KE $= \frac{1}{2}3u^2 + \frac{1}{2}1v^2$

before the collision KE $= \frac{1}{2}3 \times 2^2 + \frac{1}{2}1 \times 6^2$

The KE after the collision is 75% of the KE before, so:

$$\frac{1}{2}3u^2 + \frac{1}{2}1v^2 = \frac{3}{4}\left(\frac{1}{2}3 \times 2^2 + \frac{1}{2}1 \times 6^2\right)$$

This simplifies to:

$$3u^2 + v^2 = \frac{3}{4} \times 48 = 36$$

In the collision, linear momentum must be conserved, so taking into account the directions:

$$3u + 1v = 2 \times 3 - 1 \times 6$$

$$3u + v = 0$$

$$v = -3u$$

Substituting into the equation $3u^2 + v^2 = 36$:

$$3u^2 + (-3u)^2 = 36$$

$$12u^2 = 36$$

$$u = \pm\sqrt{3}$$

$$v = -3\sqrt{3} \text{ or } 3\sqrt{3}$$

The speed of the 1 kg sphere after the collision is $3\sqrt{3}\ \text{m s}^{-1}$

16

The answer is option **B**.

This problem can be solved by considering the displacements of point mass.

Assume that positive vector quantities are in the direction from P towards Q time $t = 0$.

The point of zero displacement is taken to be the position of P at time $t = 0$.

At time t, mass P has displacement

$$s = \tfrac{1}{2}at^2$$

$$= \tfrac{1}{2} \times 6 \times t^2$$

$$= 3t^2$$

The initial displacement of Q is $+60\,\text{m}$.

At time t, mass Q has displacement

$$s = 60 + ut + \tfrac{1}{2}at^2$$

$$= 60 + (-14t) + \tfrac{1}{2} \times 2t^2$$

$$= 60 - 14t + t^2$$

Masses P and Q meet when they have the same displacement at the same time, so

$$3t^2 = 60 - 14t + t^2$$

$$2t^2 + 14t - 60 = 0$$

$$t^2 + 7t - 30 = 0$$

$$(t - 3)(t + 10) = 0$$

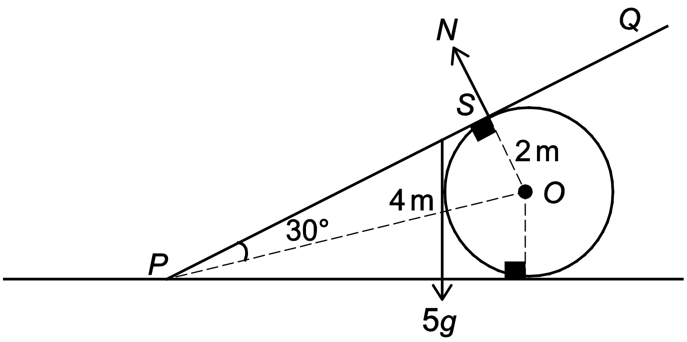
Therefore P and Q will meet when $t - 3 = 0$, so $t = 3.0\,\text{s}$.

17

The answer is option **B**.

This problem can be solved by trigonometry and taking moments.

$\sin(\text{angle } SPO) = \frac{2}{4}$, so angle $SPO = 30°$



Using Pythagoras' Theorem, $PS = \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3}$

Taking moments about P:

$$2\sqrt{3}\,N = 2\cos 60° \times 5g = 5g$$

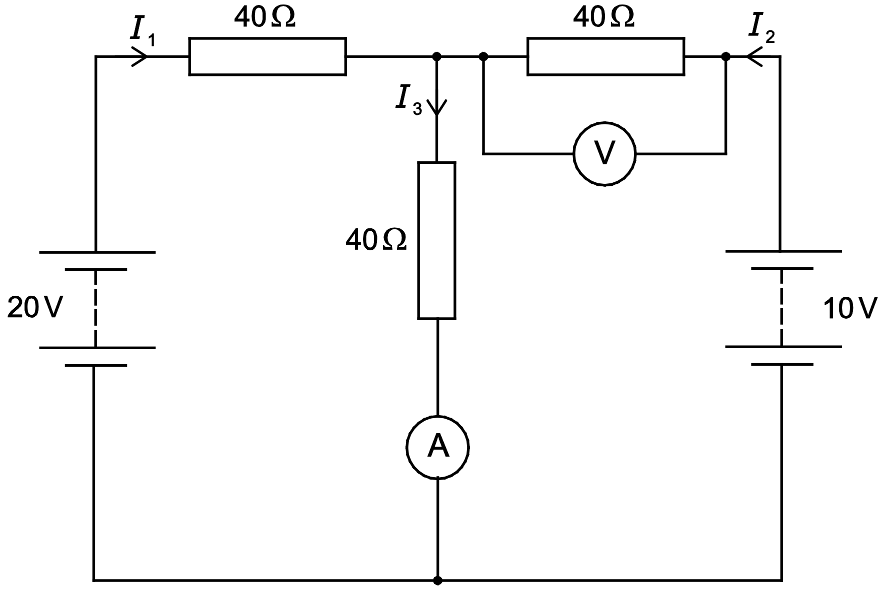
$$N = \frac{5g}{2\sqrt{3}} = \frac{5\sqrt{3}g}{6}$$

18

The answer is option **A**.

This problem can be solved using Kirchhoff's laws and solving the resulting simultaneous equations.

Label currents I_1, I_2 and I_3 as shown in the diagram.



Using Kirchhoff's laws:

$$I_3 = I_1 + I_2 \qquad (1)$$

$$20 = 40I_1 + 40I_3 \qquad (2)$$

$$10 = 40I_2 + 40I_3 \qquad (3)$$

From (1) $I_1 = I_3 - I_2$, so substitute for I_1 into (2):

$$20 = 80I_3 - 40I_2$$

Add this to (3):

$$30 = 120I_3$$

giving the ammeter reading I_3 :

$$I_3 = \frac{30}{120} = 0.25\,\text{A}$$

Substitute for I_3 into (3):

$$10 = 40I_2 + 10$$

$$I_2 = 0\,\text{A}$$

The voltmeter is therefore connected across a resistor that has no current, so the voltage across it is zero, and the voltmeter reading is 0 V.

19

The answer is option **E**.

This problem can be solved by considering both the hydrostatic pressure and the pressure due to the force of the water falling.

The water falls 45 m. Its velocity just before hitting the rock can be calculated from conservation of energy:

change in kinetic energy = change in gravitational potential energy

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 10 \times 45}$$

$$= \sqrt{900}$$

$$= 30\,\mathrm{m\,s^{-1}}$$

The pressure on the rock surface arises from two things. Firstly, the hydrostatic pressure from water resting on the surface, and secondly the pressure exerted on the surface as the water stops, equal to its rate of change of momentum divided by the area of the rock.

hydrostatic pressure $P_1 = \rho gh = 1000 \times 10 \times 0.050 = 500\,\mathrm{Pa}$

pressure $P_2 = \dfrac{F}{A} = \dfrac{\Delta(mv)}{A\Delta t} = \dfrac{40 \times 30}{2.0} = 600\,\mathrm{Pa}$

Total pressure exerted on the rock is $P = P_1 + P_2 = 500 + 600 = 1100\,\mathrm{Pa}$.

20

The answer is option **A**.

This problem can be solved by considering the circuit as a potential divider circuit, and considering possible values of the voltages.

Component X and the 200 Ω resistor are in parallel. For the pd across component X to be greater than 2.0 V, the pd across the 200 Ω resistor must be greater than 2.0 V.

The battery supplies 12 V, so the pd across the thermistor must be less than 10 V.

Using this information and the equation $\frac{V_1}{V_2} = \frac{R_1}{R_2}$, the ratio of the resistances of the thermistor and the 200 Ω resistor must be less than the ratio of the voltages across them:

$$\frac{R}{200} < \frac{10}{2}$$

$$R < 5 \times 200$$

$$R < 1000\,\Omega$$

Therefore:

$$R_o b^{-\mu T} < 1000\,\Omega$$

$$b^{-\mu T} < (1000 / R_o)$$

$$-\mu T < \log_b(1000 / R_o)$$

$$\mu T > \log_b(R_o / 1000)$$

$$\mu T > (\log_b R_o - \log_b 1000)$$

$$T > (1 / \mu)\,(\log_b R_o - \log_b 1000)$$