UNIVERSITY OF CAMBRIDGE

ECONOMICS   **D563/31**
ADMISSIONS ASSESSMENT

November 2020   **60 minutes**

SECTION 1

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts, **A** and **B**, and you should attempt **both** parts.

Part A   Mathematics (20 questions)
Part B   Advanced Mathematics (20 questions)

You are **strongly** advised to divide your time equally between the two parts: 30 minutes on **Part A** and 30 minutes on **Part B**. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 40 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 27 printed pages and 1 blank page.

            PV1

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PART A Mathematics

1   The admission charge to a cinema is different for adults and children.

Admission for 2 adults and 3 children costs £20.

Admission for 4 adults and 4 children costs £34.

What does admission cost for 6 adults and 2 children?

   A   £27

   B   £29

   C   £33

   D   £39
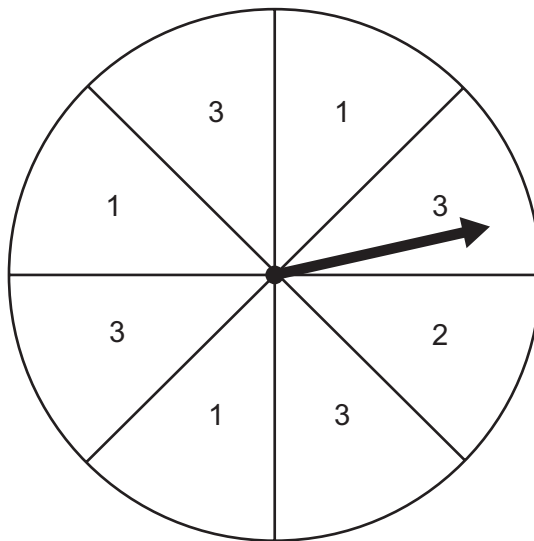
   E   £44

   F   £48

   G   £72

2   The nth term of a sequence is $2n - 5$.

Which row in the table is correct for this sequence?

	term-to-term rule	*term which has a value of 17*
A	subtract 5	11th
B	subtract 5	29th
C	subtract 2	11th
D	subtract 2	29th
E	add 5	11th
F	add 5	29th
G	add 2	11th
H	add 2	29th

3 A fair spinner has eight equal sections.
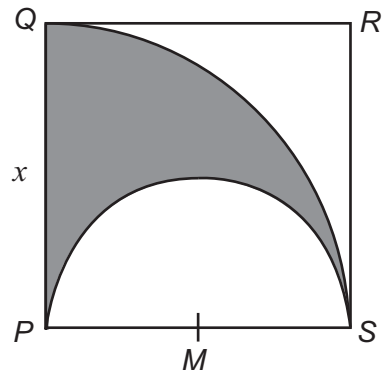
Each section has one number written on it, as shown.

The spinner is spun twice, and the two numbers scored are added.

What is the probability that the sum of the two numbers is 5?

A $\dfrac{1}{8}$

B $\dfrac{5}{8}$

C $\dfrac{1}{16}$

D $\dfrac{3}{16}$

E $\dfrac{25}{64}$

F $\dfrac{55}{64}$

4

PQRS is a square with side length x.

M is the midpoint of side *PS*.

A circular arc, with centre *M*, is drawn inside the square from *S* to *P*.

Another circular arc, with centre *P*, is drawn inside the square from *S* to *Q*.

What is the area of the shaded region in terms of x?

A $\dfrac{1}{8}\pi x^2$

B $\dfrac{3}{16}\pi x^2$

C $\dfrac{1}{4}\pi x^2$

D $\dfrac{5}{16}\pi x^2$

E $\dfrac{3}{8}\pi x^2$

F $\dfrac{7}{16}\pi x^2$

G $\dfrac{1}{2}\pi x^2$

5   A balloon contains $5000 \, \text{cm}^3$ of gas.

The gas in the balloon gradually escapes so that the volume of the balloon decreases.

60% of the volume of the balloon is lost each week.

What is the volume of the balloon, in cm^3, after 3 weeks?

A   0

B   128

C   320

D   800

E   1080

6   Consider the four lines with the following equations.

    1 $2x + 6y = 3$

    2 $9y = 3x - 4$

    3 $2y = 6x + 3$

    4 $4x + 6y - 9 = 0$

Which two lines are perpendicular?

A   1 and 2

B   1 and 3

C   1 and 4

D   2 and 3

E   2 and 4

F   3 and 4

7 The equilateral triangle *PQR* has sides of length 8 cm.

A circle, centre *O*, passes through each of the vertices of the triangle.

Find an expression for the circumference of the circle, in cm.

A $\dfrac{\sin 60°}{8\pi}$

B $\dfrac{8\pi}{\sin 60°}$

C $\dfrac{\cos 60°}{8\pi}$

D $\dfrac{8\pi}{\cos 60°}$

E $\dfrac{\tan 60°}{8\pi}$

F $\dfrac{8\pi}{\tan 60°}$

8 Find the sum of the solutions of

$$2\left(\frac{x}{4}+3\right)^2 - \left(\frac{x}{4}+3\right) - 36 = 0$$

A 2

B $\dfrac{3}{2}$

C $\dfrac{1}{2}$

D −4

E −13

F −22

G −26

H −34

9 When the expression

$$(2x + 3)^2 - (x - 3)^2$$

is written in the form $p(x + q)^2 + r$, where p, q and r are constants, what is the value of r?

A -27

B -9

C 0

D 3

E 15

10 Which one of the following expressions is equivalent to

$$\frac{a}{b/c} - \frac{a/b}{c}$$

A 0

B $\dfrac{a(b^2 - 1)}{bc}$

C $\dfrac{a(b^2 - c^2)}{bc}$

D $\dfrac{a^2 b^2 - c^2}{abc}$

E $\dfrac{a(c^2 - 1)}{bc}$

F $\dfrac{a^2 c^2 - b^2}{abc}$

G $\dfrac{b^2 - a^2}{abc}$

11 The table shows statistics relating to the test marks of two groups of students.

	number of students	mean	range
group X	10	36	16
group Y	20	48	21

The results for the two groups of students are combined.

What can be deduced about the mean and range of the combined results?

A mean $= 40$, range ≤ 16

B mean $= 40$, $16 <$ range < 21

C mean $= 40$, range ≥ 21

D mean $= 44$, range ≤ 16

E mean $= 44$, $16 <$ range < 21

F mean $= 44$, range ≥ 21

12 The number of pairs of winter boots sold on a day is inversely proportional to the cube of the outside temperature on that day, measured in °C.

On a day when the outside temperature is $8\,°C$, 250 pairs of boots are sold.

The next day, when the outside temperature is $x\,°C$, the number of pairs of boots sold is 700% more than on the previous day.

What is the value of x?

A 2

B 4

C $\dfrac{8}{\sqrt[3]{7}}$

D $8\sqrt[3]{7}$

E 16

13   In a sale, all prices are reduced by 25%.

A customer calculates the pre-sale price of a bicycle incorrectly by increasing the marked sale price by 25%.

The customer's calculated pre-sale price is incorrect by £15.

What is the correct pre-sale price of the bicycle?

A   £180

B   £195

C   £210

D   £225

E   £240

14   A paint colour is a mixture of red paint, blue paint and yellow paint.

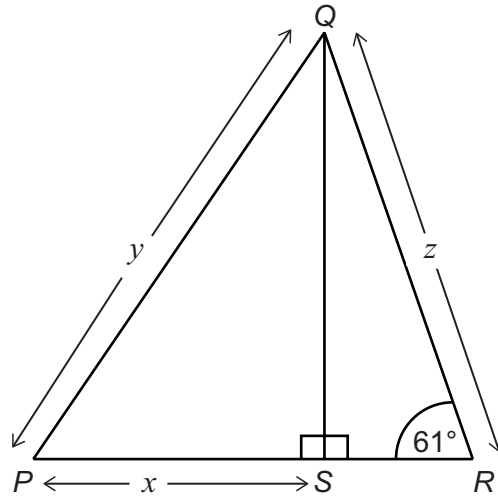The ratio of red paint to blue paint in the mixture is $18:5$

The ratio of blue paint to yellow paint in the mixture is $p:3$

The ratio of red paint to yellow paint in the mixture is $12:5$

What is the value of p?

A   2

B   4.5

C   5

D   7.5

E   12

15

[diagram not to scale]

In the diagram, *QS* is perpendicular to *PR*.

PS = x cm
PQ = y cm
QR = z cm

angle *QRS* = 61°

PSR is a straight line.

Which one of the following is an expression for the length z, in cm?

A $\sqrt{y^2 + x^2}\,\sin 61°$

B $\sqrt{y^2 - x^2}\,\sin 61°$

C $\sqrt{y^2 + x^2}\,\cos 61°$

D $\sqrt{y^2 - x^2}\,\cos 61°$

E $\dfrac{\sqrt{y^2 + x^2}}{\sin 61°}$

F $\dfrac{\sqrt{y^2 - x^2}}{\sin 61°}$

G $\dfrac{\sqrt{y^2 + x^2}}{\cos 61°}$

H $\dfrac{\sqrt{y^2 - x^2}}{\cos 61°}$

16 Two identical fair six-sided dice each have their faces numbered from 1 to 6, with one number on each face.

Both dice are thrown, and the number on each of the dice is recorded.

They are then both thrown again, and the number on each of the dice is recorded.

What is the probability that at least one of the four recorded numbers is even?

A $\dfrac{1}{4}$

B $\dfrac{1}{2}$

C $\dfrac{9}{16}$

D $\dfrac{3}{4}$

E $\dfrac{15}{16}$

17 The quadratic equation $2x^2 - px - 4 = 0$, where p is a positive constant, has two solutions that differ by 6.

What is the value of p?

A 2

B $4\sqrt{7}$

C 12

D $4\sqrt{11}$

E $4\sqrt{34}$

F $6\sqrt{30}$

18 Two vertices of a square are at $(1, 1)$ and $(3, 5)$.

What is the difference between the perimeters of the largest and smallest possible squares that can be drawn with these points as two of their vertices?

A   0

B $4\sqrt{3}\left(2 - \sqrt{2}\right)$

C $4\sqrt{3}\left(\sqrt{2} - 1\right)$

D $4\sqrt{5}\left(2 - \sqrt{2}\right)$

E $4\sqrt{5}\left(\sqrt{2} - 1\right)$

F $4\sqrt{13}\left(2 - \sqrt{2}\right)$

G $4\sqrt{13}\left(\sqrt{2} - 1\right)$

H $4\sqrt{3}\sqrt{5}\left(2 - \sqrt{2}\right)$

19 The point *M* is (2, 5) and the point *N* is (−3, −1).

The line segment *MN* is transformed to the line segment *TU* by two transformations:

MN is rotated 90° clockwise about the origin to give the line segment *RS*.

RS is then translated by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$ to give the line segment *TU*.

The coordinates of the midpoint of *TU* are (7, −2.5).

Find the vector $\begin{pmatrix} p \\ q \end{pmatrix}$

A $\begin{pmatrix} 2 \\ 0.5 \end{pmatrix}$

B $\begin{pmatrix} 0.5 \\ 2 \end{pmatrix}$

C $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

D $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$

E $\begin{pmatrix} 9 \\ -2 \end{pmatrix}$

F $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$

20 A solid cone has a base radius x cm.

The ratio of the perpendicular height of the cone to the radius of the cone is $5 : 2$

A solid hemisphere of radius $\dfrac{y}{2}$ cm is made from the same material as the cone.

Which one of the following is a correct expression for

$$\frac{\text{volume of the cone}}{\text{volume of the hemisphere}}$$

(Volume of a cone $= \frac{1}{3}\pi r^2 h$ where r is the radius and h is the perpendicular height.)

(Volume of a sphere $= \frac{4}{3}\pi r^3$ where r is the radius.)

A $\dfrac{5x^3}{y^3}$

B $\dfrac{5x^3}{4y^3}$

C $\dfrac{8x^3}{5y^3}$

D $\dfrac{10x^3}{y^3}$

E $\dfrac{14x^3}{y^3}$

PART B Advanced Mathematics

21 Given that

$$\int_4^9 f(x)\,\mathrm{d}x = 3$$

and that

$$g(x) = 2f(x) + 1$$

find

$$\int_4^9 g(x)\,\mathrm{d}x$$

A 6

B 7

C 8

D 9

E 10

F 11

22 $(x - 1)$ and $(x - 2)$ are both factors of $x^4 + ax^3 + bx^2 - 12x + 4$
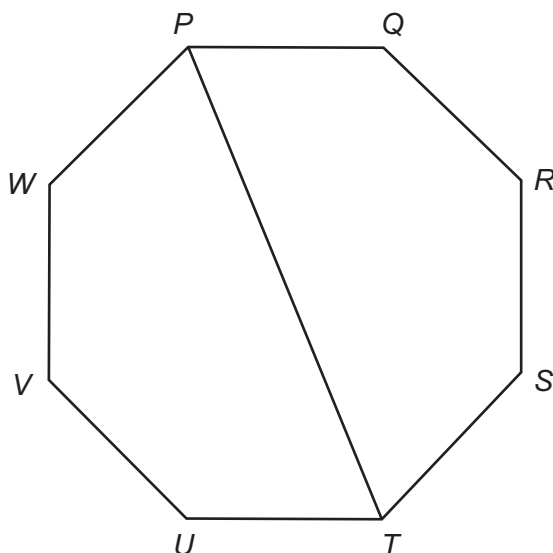
What are the values of a and b?

A $a = -6$ and $b = -23$

B $a = -6$ and $b = 13$

C $a = 6$ and $b = -11$

D $a = 6$ and $b = 1$
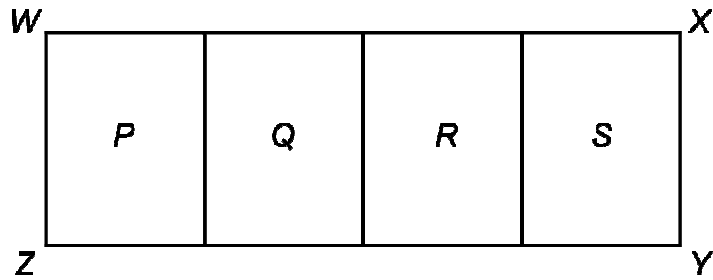
23

The area of the regular octagon is $32\sqrt{2}$ cm^2.

What is the length, in cm, of the straight line *PT*?

A   8

B   16

C $4\sqrt{2}$

D $8\sqrt{2}$

E $4\sqrt{2\sqrt{2}}$

F $16\sqrt{2\sqrt{2}}$

24   What is the area of the region enclosed between the curve $y = \frac{1}{2}x^2$, the line $y = -x$, and the lines $x = 1$ and $x = 3$?

A $\dfrac{1}{3}$

B   2

C   4

D   6

E $\dfrac{25}{3}$

F $\dfrac{28}{3}$

25

A wall is used to mark out the perimeter of a rectangular field *WXYZ*.

Walls are also used to divide *WXYZ* into four identical rectangular fields, *P*, *Q*, *R* and *S*, as shown.

The total length of wall used is 260 m.

What is the length of *WZ* that maximises the area *P*?

A   20 m

B   26 m

C   32.5 m

D   40 m

E   52 m

F   65 m

26   A line with non-zero gradient m is reflected in the line $y = x$

What is the gradient of the reflected line?

A m

B $-m$

C $\dfrac{1}{m}$

D $-\dfrac{1}{m}$

27 There are two red balls and two blue balls in a bag.

Two balls are removed at random without replacement.

Given that at least one of them is red, what is the probability that one of them is blue?

A $\dfrac{1}{2}$

B $\dfrac{2}{3}$

C $\dfrac{4}{5}$

D $\dfrac{5}{6}$

E 1

28 The sum of the first 20 terms of an arithmetic progression is 50.

The sum of the next 20 terms of the arithmetic progression is -50.

What is the sum of the first 100 terms of the arithmetic progression?

A -750

B -350

C -50

D $-\dfrac{159}{8}$

E $\dfrac{159}{8}$

F 50

G 350

H 750

29 A sequence is generated by

$$x_{n+1} = -\frac{12}{x_n}$$

n is an integer, where $n \geq 1$.

The 50$^{\text{th}}$ term of the sequence is 6.

What is the sum of the first fifteen terms of the sequence?

A -2

B   6

C   10

D   22

E   26

F   34

G   58

30 The line L with equation $y = mx + c$, where $m > 0$ and $c \geq 0$, passes through the point $(2, 4)$.

A line is drawn through the point $(2, 4)$ perpendicular to L.

The triangle enclosed between the two lines and the y-axis has area 5 square units.

What is the **larger** of the two possible values of m?

A -0.5

B   0.5

C   1.25

D   2

E   5

31 A list of n numbers has mean m and a unique mode d.

Two numbers are removed from the list.

The remaining list of numbers also has a unique mode, but this unique mode is not equal to d.

The mean of the remaining $n - 2$ numbers is $m + 2$.

What was the unique mode, d, of the original list?

A $n - m + 2$

B $n - m - 2$

C $n + m - 2$

D $m + n + 2$

E $m - n + 2$

F $m - n - 2$

32 P and Q are two different geometric progressions.

The 3rd term of each geometric progression is 4.

The 5th term of each geometric progression is 2.

What is the modulus of the difference between the sums to infinity of P and Q?

A 0

B 8

C $8\sqrt{2}$

D 16

E $16\sqrt{2}$

F 32

G $32\sqrt{2}$

33 Two circles have centres P and Q.

The radius of each circle is 1 cm.

The distance PQ is 1 cm.

What is the area of overlap, in cm^2, of the two circles?

A $\quad \dfrac{\pi}{3} - \dfrac{1}{4}$

B $\quad \dfrac{\pi}{3} - \dfrac{\sqrt{3}}{4}$

C $\quad \dfrac{2\pi}{3} - \dfrac{1}{2}$

D $\quad \dfrac{2\pi}{3} - \dfrac{\sqrt{3}}{2}$

E $\quad \dfrac{4\pi}{3} - \dfrac{1}{4}$

F $\quad \dfrac{4\pi}{3} - \dfrac{\sqrt{3}}{2}$

34 The curve

$$y = x^3 + 3\sqrt{5}\,px^2 + 3px + 13$$

has two distinct turning points.

What are all the possible values of p?

A $\quad p < 0$, $p > 0.2$

B $\quad p \leq 0$, $p \geq 0.2$

C $\quad 0 < p < 0.2$

D $\quad 0 \leq p \leq 0.2$

E $\quad p < 0$, $p > 1.2$

F $\quad p \leq 0$, $p \geq 1.2$

G $\quad 0 < p < 1.2$

H $\quad 0 \leq p \leq 1.2$

35 Find the complete set of values of x for which

$$x^4 + 36 < 13x^2$$

A $4 < x < 9$

B $x < 4$, $x > 9$

C $-9 < x < -4$, $4 < x < 9$

D $x < -9$, $-4 < x < 4$, $x > 9$

E $2 < x < 3$

F $x < 2$, $x > 3$

G $-3 < x < -2$, $2 < x < 3$

H $x < -3$, $-2 < x < 2$, $x > 3$

36 Find the number of solutions of the equation

$$14\cos^3 x + 10\sin^2 x \cos x = 13\cos x$$

in the range $-2\pi \le x \le 2\pi$

A 4

B 6

C 8

D 10

E 12

F 14

37 It is given that

$$2^x = 3^y$$

and

$$x + y = 2$$

Which one of the following is an expression for x?

A $2\log_{10}\dfrac{1}{2}$

B $\log_{10} 3$

C $\dfrac{6}{5}$

D $\dfrac{3}{2}$

E $\dfrac{\log_{10} 9}{\log_{10} 5}$

F $\dfrac{\log_{10} 9}{\log_{10} 6}$

G $\dfrac{\log_{10} 3}{\log_{10} 2}$

38 Find the product of the real roots of the equation

$$\left(\log_{10} x^2\right)^2 + \log_{10} x = 3$$

A $10^{-\frac{3}{2}}$

B 10^{-1}

C $10^{-\frac{1}{2}}$

D $10^{-\frac{1}{4}}$

E $10^{\frac{3}{5}}$

F 10^{1}

39 Find the y-coordinate of the points on the curve $y = x^2$ that are closest to the point $\left(0, \dfrac{9}{2}\right)$

    A   0

    B $\dfrac{1}{4}$

    C $\dfrac{4}{3}$

    D   4

    E $\dfrac{9}{2}$

40 Find the maximum value of the gradient of the curve with equation

$$y = 2 - 4x + 4x^{\frac{3}{2}} - x^2$$

where $x > 0$

    A -4

    B $-\dfrac{8}{9}$

    C $\dfrac{1}{2}$

    D   2

    E   4

END OF TEST

TZ3	Answer_key
Q1	D
Q2	G
Q3	A
Q4	A
Q5	C
Q6	B
Q7	B
Q8	F
Q9	A
Q10	E
Q11	F
Q12	B
Q13	E
Q14	A
Q15	F
Q16	E
Q17	B
Q18	D
Q19	C
Q20	D
Q21	F
Q22	B
Q23	A
Q24	E
Q25	B
Q26	C
Q27	C
Q28	A
Q29	E
Q30	D
Q31	E
Q32	E
Q33	D
Q34	A
Q35	G
Q36	E
Q37	F
Q38	D
Q39	D
Q40	C

UNIVERSITY OF CAMBRIDGE

ECONOMICS
ADMISSIONS ASSESSMENT

D563/32

November 2020 SECTION 2

60 minutes

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so. This paper is Section 2 of 2.

This question paper requires you to read a single passage and complete a related task.

You should write your answer in the space provided in this question paper. Please complete this section in **black pen.**

You can use the blank inside front and back covers for rough working or notes, but no extra paper is allowed. Only answers in the space indicated in the paper will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 10 printed pages and 2 blank pages.

PV1

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Read the article 'Wealth taxes have moved up the political agenda' taken from *The Economist* (October 3rd 2019). Based on this text, complete the task below.

TASK

- **Set out one reason why a wealth tax might discourage investments that result in technological innovations and one reason why such a tax might encourage them.**

- **Discuss the broader advantages and disadvantages of introducing a wealth tax in a rich developed economy.**

Your answer will be assessed taking into account your ability to construct a reasoned, insightful and logically consistent argument with clarity and precision.

Wealth taxes have moved up the political agenda

Some economists are reconsidering their aversion to levies on large fortunes

Five years ago Thomas Piketty's "Capital in the Twenty-First Century", a weighty analysis of rising inequality, flew off shelves and ignited fiery debate. Fans and detractors alike tended to agree on one thing, at least: its proposal to fix inequality – a tax on wealth – was a dud. A half-decade later the mood has shifted. Several candidates for the Democratic presidential nomination promise to tax wealth; Bernie Sanders recently announced a plan to tax fortunes of more than $32m at 1% per year, and those larger than $10bn at 8%. In his latest doorstopper, "Capital and Ideology", currently available only in French, Mr Piketty suggests taxing the wealth of billionaires at up to 90%. Few economists go so far. But more are now arguing that wealth taxes need not slow growth.

The shifting political climate is not hard to explain: taxes on wealth are popular. An analysis of recent survey evidence, for example, found that Americans favour such levies, especially on inheritance. And the case for taxing wealth has become easier to make. Emmanuel Saez and Gabriel Zucman of the University of California, Berkeley, find that the top 0.1% of taxpayers accounted for about 20% of American wealth in 2012, up from 7% of wealth in 1978 and close to levels last seen in 1929. The vast fortunes of the very rich – for example the more than $100bn controlled by Jeff Bezos, the founder and boss of Amazon – make juicy targets, too, for politicians seeking to fund new spending.

Economists have long been hostile to wealth taxes. But not Mr Piketty, Mr Saez or Mr Zucman. Mr Piketty based his case on the argument that concentrated wealth leads to concentration of political power, which undermines democracy. Mr Saez and Mr Zucman agree, and cite other concerns. In a recent paper, for instance, they note that in America the ratio of household wealth to national income has nearly doubled over the past 40 years, mostly because of the rising value of assets. Higher asset values could mean that firms are becoming more efficient – or it could reflect economic sclerosis. Property values could be rising because regulations make it difficult to build, for instance, and higher stock prices could be a sign that markets are becoming less competitive, and profits thus easier to come by. Taxing and redistributing wealth, then, could be a justified response to misfiring markets.

Other economists are warming to the idea. In a new paper published by the National Bureau of Economic Research, a team of five economists aims squarely at the standard economic argument against wealth taxes. Today's wealth is yesterday's income, that reasoning goes, so wealth taxes are bad because they discourage income-generating activities, such as work and investment. Taxes on capital in particular should be spared, because investment is an input into future growth. Taxes that discourage investment mean less output today and a smaller economy tomorrow. In some economic models the optimal tax on capital is a whopping 0%.

But these models often assume that one investment is as good as the next. In practice, say the authors of the new paper, that is far from true. Some people stash their money in low-yield government bonds; others fund startups that become trillion-dollar companies. Shifting the burden of tax from capital income to wealth, they argue, would reward investors capable of achieving outsize returns on their investments, and shrink the fortunes of those unwilling or unable to put their lucre to productive use. Heirs would feel pressure to use their wealth or lose it. Entrepreneurs accustomed to achieving double-digit returns would scarcely notice a modest wealth tax. Designed well, the authors reckon, it could reduce inequality while raising productivity.

The authors' use-it-or-lose-it approach to wealth taxation has some similarities with arguments for taxes on land values (which this newspaper favours). Henry George, a 19th-century American journalist, became the Thomas Piketty of his day by campaigning for such levies. The rents earned by wealthy landowners derive in part from improvements they make to the land, he argued, but also from land's scarcity. A land-value tax collects on behalf of society the value attributable to the land itself, while leaving owners to collect the returns on investments in the land, such as buildings, untaxed. Similarly, shifting the burden of tax from capital income to wealth rewards ongoing efforts to deploy money well.

Economists like land-value taxes because they are efficient. But they also have a certain moral appeal. Society sets the terms on which individuals can accumulate wealth. It makes sense to structure those terms to benefit society as a whole. Wealth taxes are often cast as punitive – an impression encouraged by supporters, like Mr Sanders, who believe that "billionaires should not exist". But designed well, a wealth tax could confer greater moral legitimacy on large fortunes, because keeping them means continually putting them to productive ends.
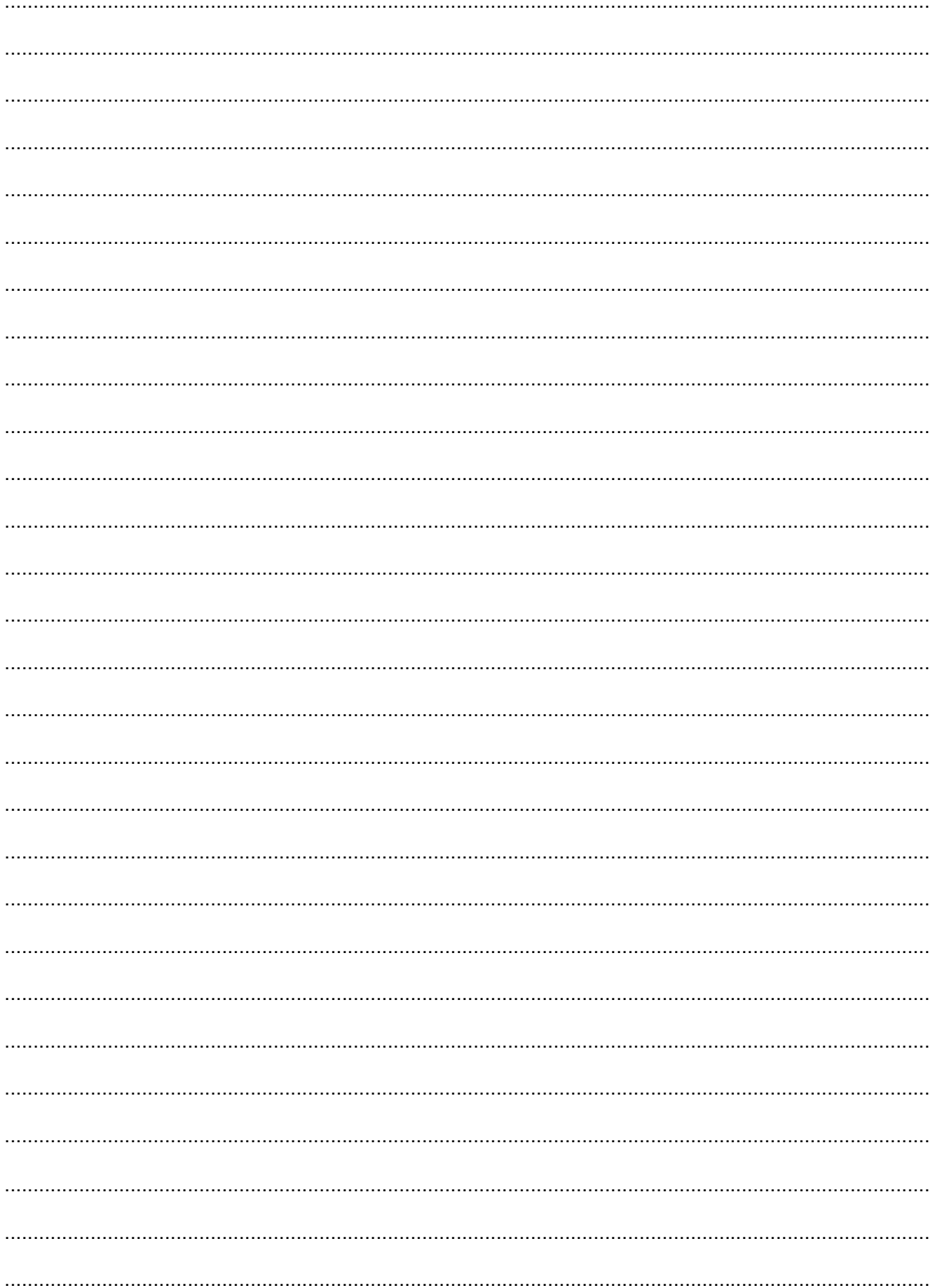
All's well that ends wealth

Wealth taxes have their complications. Defining what kinds of investment are more productive than others is difficult. Instead of encouraging more risk-taking they might encourage tax avoidance – and emigration, since the rich are often highly mobile. In Europe, where citizens can easily move country and policing of tax evasion is lax, wealth taxes have been hard to sustain. But some politicians reckon that the challenges are surmountable. Elizabeth Warren, another Democratic presidential contender, would hit Americans who renounce their citizenship for tax purposes with an "exit tax" of 40% of their net worth above $50m. Financial institutions maintain detailed information on clients' wealth balances; governments could require them to share this information with tax authorities. Governments' patience with tax havens, already waning, could fail entirely if wealth taxation spreads.

Overshoot is clearly a risk. An energised American left, if elevated to power, could easily go too far. But wealth taxes are not necessarily an affront to economics. They are worth debating.

Write your answer in the space below.

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